THE PROBABILITY OF DAMAGE TO AN AIRCRAFT CARRIER FROM AN UNDERWATER EXPLOSION

by

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ABSTRACT

This thesis presents a method, through the use of probability theory, of determining specific types of damage to an aircraft carrier. Consideration is given to the cases of loss of main machinery spaces and shafting resulting from side contact and underbottom non-contact explosions against a carrier possessing an underwater protective system.

Probability density functions for the distribution of target hits, damage length, and charge weights are proposed. These are then incorporated into integral expressions which when evaluated within prescribed limits will determine the appropriate probabilities of damage.

Illustrative examples of some of the proposed results are presented as a check on the validity of the theoretical expressions. Recommendations are made for means of obtaining actual density functions to insure completeness of this method of probabilistic approach to the determination of damage to a ship.

Thesis Supervisor: Edward S. Arentzen

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I. INTRODUCTION

With the design of ships of the U. S. Navy, an important consideration is the ability of the ship to withstand severe damage and still continue to fight. This resolves itself into the capabilities of the protection features incorporated into the design. That this is important is shown by the fact that statements on protection requirements are included in the approved ship characteristics. Thus ship protection can be considered a military characteristic defined as comprising those features in a ship which nullify or minimize weapons effects and permit continued effective operations for the maximum time.[(1)*]

Naturally the ship designer must concern himself with the protection features needed to withstand attacks in terms of all categories, such as from underwater explosions, air blast, ballistic penetration, and radiological and toxicological. However the subject matter contained herein will be limited to dealing with the protection of the ship from underwater explosions.

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* Numbers in parentheses refer to references at the end of this paper.

The effects of underwater explosions on ships have been studied ever since this type of attack became included in the field of naval warfare. Although the first tests against ships were conducted in the late nineteenth century, it was not until prior to World War I that extensive test programs were conducted by all major naval powers.[2] For the United States, the greatest work done in this field at that time was that accomplished by Admiral Stocker. Studies of the effects of underwater explosions were started by the U.S. Navy about 1907, with the systematic work of Admiral Stocker being published in 1922. This work provided a sound basis for further improvements in side protection systems and has formed the basis for progress which continues even up to the present day.[1]

The interval between World War I and World War II brought about efforts to develop and improve the design of special hull arrangements to reduce the effects of underwater explosions. This was accomplished through the testing of various designs of ship sections or their models under different severities of attack. World War II saw a great intensification of the research on underwater explosions, which has continued since the war, especially in the U. S. Navy.[2]

A great deal of the information obtained as a result of these tests is presented and evaluated in references

15 through 22. Because of their military significance, these references are all classified. However since this thesis is not an attempt to discuss the development of design principles from background material or test results, it is not essential to utilize this information as such. Fortunately some general statements can be made about underwater protection systems in an attempt to describe for the uninformed reader some of the problems faced by the naval designer in this field. In this way it is the authors' hope to justify their undertaking the problem of determining if there is a simple format that can be used by the designer to evaluate a particular protective system.

For example the designer of a side protective system is of necessity faced with rigid limitations as to weight and space. Thus he is sometimes required to increase the amount of protection with only slight variations over a previous design. Fortunately the research mentioned previously has resulted in the applications of new steels and structural arrangements, as well as a better understanding of the phenomena associated with underwater explosions. This in turn has resulted in the progress of the side protective system to a high degree of refinement. Thus the present-day FORRESTAL class of aircraft carrier can defeat torpedoes of significantly larger size than a World War II

battleship or a MIDWAY class carrier. And this capability has been achieved at a very small increase of weight per square foot of transverse depth.(1)

However the progress in protection against an under-bottom explosion has not been as rapid since for a given charge weight there is a relatively more severe nature of loading. More important, though, is the heavy price that is paid when machinery weights are raised as a means of providing more depth in the bottom. Several possible methods of further improving damage resistance to under-bottom explosions are being explored, such as distribution of structural weight and liquid loading in the bottom.

Thus there is no simple approach to the problem of underwater protection. Even after years of explosion testing and useful research, many aspects of protective system design are complex and defy exact analysis. A great deal of data has been accumulated and repeatedly analyzed, although a general over-all theory covering all phases of the subject has not been formulated. This is because any protective system design must be a compromise between the charge weight it can defeat and the space and weight allotted to the system. Thus within the allowable weight, the protection system should reject the maximum amount of explosive energy, limit as much as

possible the extent of damage, and retain sufficient strength for the ship to continue to operate.

Thus the attainment of maximum practical damage resistance must be a design objective for naval ships. Therefore it would seem to be worthwhile if a useful, theoretical format can be developed in order to allow a designer to evaluate the relative merit of several different protection systems and space arrangements in regard to types of damage in order to obtain the optimum system. In this respect, this paper will deal with determining such a format using probability theory. Because the problem of developing such a format for an entire ship's protection system and space arrangements would be impossible in the time available, this investigation must be limited in scope. With this in mind, the authors decided to study only the area of the machinery spaces of a large aircraft carrier and their side and underbottom protection systems. This in no way implies that these spaces were considered to be the only essential ones of the ship. Certainly other spaces, such as magazines and gasoline tanks, would require similar protection. But it was felt that if the format could be developed for the case of the machinery spaces, it could be later extended so as to include other vital spaces.

Thus we shall attempt to aid in the study of the overall problem by solving a small section of it. Although this may be considered unrealistic, it is hoped that such an approach can determine the feasibility of the naval designer using probabilistic methods in evaluating the type and amount of damage to be expected from an underwater explosion occurring against or in proximity to different ships having specific arrangements and protection systems.

II. PROCEDURE

The procedure to be followed in this paper is to determine for the case of an aircraft carrier possessing an underwater protection system the probability of a specific type of damage, that is, the loss of machinery spaces and shafts, which result from an underwater explosion. This damage which the ship will experience is a function of many variables, such as the charge weight, the longitudinal location of the explosion, the arrangement of machinery spaces and transverse bulkheads, the types of protection systems, and, in regard to an underbottom explosion, the speed of the ship and the standoff distance of the explosion. Therefore the first step in this probabilistic approach is to determine the appropriate density functions to be used for each of these variables.

In this respect, some reference is made to existing data if available, (see Appendix C). Otherwise assumptions are made to determine what form the applicable probability density functions may have. It must be stated that usually for any theoretical investigation, a large number of basic assumptions is necessary. The

more these assumptions are simplied, the easier becomes the mathematical treatment, but the less valuable become the results. Therefore a proper sense of values must be maintained, with the assumptions used being such as to correctly typify the characteristics of the problem while reducing the mathematical difficulties to a minimum.

With these density functions determined, the probabilities of damage (loss of shafts and machinery spaces) are developed. In this respect, the following section of the paper is divided into two parts. Part A deals with the case of the underwater side contact explosion and its related probabilities of loss of spaces and shafts expressed in general form. Part B discusses the case of the underbottom non-contact explosion and its associated probabilities.

Illustrative examples of some of the calculations for several probabilities of damage for each of the cases mentioned above will be presented in Appendices A and B. In addition, the adaptability to computer programming of the format to be developed will be discussed in Appendix D. Also a brief discussion will be given in Appendix E to show how this method of probabilistic approach might be used to evaluate the effectiveness of combined versus separate machinery spaces on large ships.

III. RESULTS

A. Underwater Side Contact Explosion

When a torpedo explodes in contact with the shell of a ship it tears a large hole in the fore-and-aft direction into the ship's hull. Close to the point of attack, the rupture of the longitudinal or transverse bulkheads can be due either to direct exposure to the explosion forces, or to deformation caused in the bulkheads by hull deformation. Fragmentation of the shell in the area of the impact creates additional hazards since the fragments travel at high velocities and may pierce the inner bulkheads and hence extend the damage area. Of course the major damage occurs because the large hole causes the compartment to flood rapidly with the resulting loss of the space. If the bulkheads are damaged severely, the flooding can spread rapidly into adjacent compartments. Thus the damage in the immediate attack area is devastating, with the extent depending on the size of the charge.[2] If, as in the cases to be considered here, we are concerned only with machinery spaces, this loss of a space will also result in the loss

of the shaft which originates in the space.

It may be noted at this point that because the energy release from a contact explosion of a mine or torpedo is of such large magnitude, the defense efforts have not been aimed at total rejection of the attack. Instead, design considerations have been based on a combination of rejection by structural strength, absorption through optimum use of plastic deformation of structure, limitation of extent by strong boundaries, and absorption-rejection through air compression. Studies have been made concerning the matter of optimum liquid loading in the protective system from such considerations as fragment penetrations, and best compromise between damage prevention and damage control for location and thickness of layer.[1] Thus it is important that exact design of the side protection system be known beforehand in order to obtain, for the equation to be developed, an accurate value of the probability density function of the damage to be expected from a certain weight of charge.

In this part of the thesis we shall consider the case of an aircraft carrier and investigate the situation involving a single side contact hit. We shall assume that the carrier is of present day design possessing a multi-

bulkhead longitudinal side protective system capable of defeating charges below a certain weight. The machinery spaces are arranged as shown in Figure I, which also indicates the coordinate system that will be used through the paper. For the present we will assume that all machinery is located in these four spaces and that no separate auxiliary spaces exist. Later we will investigate the effect of introducing auxiliary spaces.

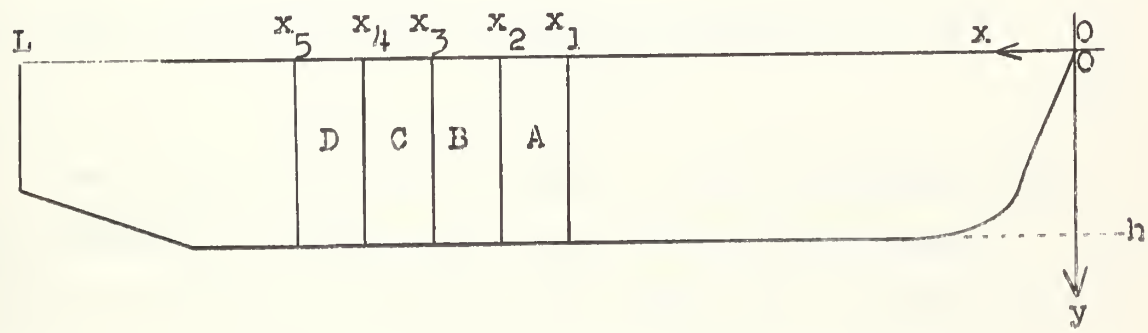


FIGURE I

Arrangement and Designation of Machinery Spaces and Coordinate System

The damage of interest for this portion of the problem is that of the loss of propulsion machinery spaces from a single side contact hit. We initiate the problem by considering the probability of the loss of a single space.

1. Case of Loss of a Particular Space, A.

A machinery space is said to be lost when the holding bulkhead of the space is ruptured or penetrated by the effects of the explosion. The probability that space A is lost is therefore equal to the probability that the holding bulkhead of space A is ruptured. The following notation will be used to denote such a probability and likewise for any similar type expression.

$$P_r \text{ (loss of space A)} = P_r \text{ (holding bulkhead of A is ruptured)} \qquad (1)$$

The holding bulkhead of space A can be ruptured under two different circumstances. One is for the attack to occur at the shell enclosing the space (i.e., within the range x_1 to x_2 of Figure I) resulting in the rupture of the holding bulkhead of A. The other is for the attack to occur in the region forward or aft of the space (between O and x_1 or between x_2 and L) with the result that the damage is sufficiently great to extend into the holding bulkhead of A. This is the case of either of two events occurring which from basic probability theory is equal to the sum of the probabilities of occurrence of each minus the probability of the simultaneous occurrence of both. Obviously for the case in question both events cannot

occur simultaneously since we have postulated a single hit. This hit cannot occur at A and adjacent to A at the same time. We will assume that the mechanism causing the loss of a space is a damage length or hole resulting from the explosion. Furthermore we will state that the damage length is symmetric about the point of impact extending equal amounts on either side of this point. This may not be true for all cases but for the sake of simplicity we shall assume that symmetry does exist.

With these assumptions Equation (1) can be stated as follows:

$$P_r(\text{loss of A}) = P_r(\text{hit occurs at A and a damage length greater than zero results})$$
$$+ P_r(\text{hit occurs adjacent to A and the damage length extends into A}) \qquad (2)$$

In order to obtain a solution for the above expression one requires a knowledge of the joint probability distribution for the pertinent events. Consider the first term on the right of Equation (2). The two events of interest are that a hit occurs at A and that a damage length in the holding bulkhead results. A hit at A can occur anywhere along x in the range x_1 to x_2 and along y in the range 0 to h. If in this range the side protective system

were not uniform throughout but varied somehow (structurally or because of liquid loading) then the two events could not be considered independent. Different damage length could occur depending upon where the hit occurred along x and y. For such a case one would need a density function relating the positional coordinates x and y, to the damage coordinates b and w, where we have chosen to define b as the length of damage and w as the charge weight. Surely damage length will depend upon the charge weight as well as the structure of the system. This function, which will be designated $f(x,y,b,w)$ could be integrated over appropriate limits to produce the desired probability. If such an expression could be found that would be valid for any x,y,b or w, then Equation (2) could be placed in integral form and would appear as shown in Equation (3).

$$
\begin{aligned}
P_r(\text{loss of A}) = & \int_0^{w_{max}} dw \int_0^{h} dy \int_{x_1}^{x_2} dx \int_0^{b_{max}} db \quad f(x,y,b,w) \\
& + \int_0^{w_{max}} dw \int_0^{h} dy \int_0^{x_1} dx \int_{2(x_1-x)}^{b_{max}} db \quad f(x,y,b,w) \\
& + \int_0^{w_{max}} dw \int_0^{h} dy \int_{x_2}^{L} dx \int_{2(x-x_2)}^{b_{max}} db \quad f(x,y,b,w) \qquad (3)
\end{aligned}
$$

The limits on b for a hit in the range of x from 0 to x_1 are such that the half damage length, b/2, must be greater than the distance of the explosion from x_1; i.e., $\frac{b}{2} > x_1 - x$ or $b > 2(x_1 - x)$. This follows from the fact that we have assumed a damage length symmetric about the point of impact.

A similar argument will lead to the limits on b for the range of x from x_2 to L.

The general expression $f(x,y,b,w)$ is difficult to realize because of its four dimensional nature. However, certain assumptions can be made which will enable one to expand the function into a product of terms relating independent events.

First of all the fact that a hit does occur at a position x is certainly independent of what else may happen because of the hit. The same reasoning holds for the position of the hit in the y direction. Similarly the choice of any weight of charge is arbitrary and independent of all other parameters. We are still left with the determination of b, the damage length. It has been previously stated that b may definitely be a function of x, y and w, but for any x, y and w we should expect b to have a certain value, or more probably a restricted range of values. Before stating anything further about b, the

above arguments enable us to expand $f(x,y,b,w)$ into a product of independent density functions.

$$f(x,y,b,w) = p(x)\ p(y)\ p(w)\ p(b/x,y,w) \qquad (4)$$

In Equation (4) $p(x)$, $p(y)$, and $p(w)$ are respectively the density functions for the distribution of hits in the x direction, the distribution of hits in the y direction, and the distribution of charge weights.

The function $p(b/x,y,w)$ is the distribution of damage lengths for a particular side protection system, given the position of the hit, x and y, and the weight of charge, w. If we make further assumptions, such that the protection system is uniform throughout the possible range of x and y, then b is only a function of w. For the scope of our problem, where we limit ourselves to the region of the main machinery rooms, then the system is uniform in x for all practical purposes. However, we know the system changes with depth because of the physical requirement of terminating the system. The ship may be able to defeat a lesser weight charge if the hit occurs at deep rather than shallow drafts.

The determination of b as a function of y and w would pose quite a task. The assumption of a protective system uniform in depth would facilitate the problem and would

not seriously detract from the overall problem. The method of approach is not altered. Furthermore one may consider that the system is uniform in depth but constructed so that it is capable of defeating a charge weight corresponding to the mean charge weight of the original system. With these assumptions it follows that $p(b/w,x,y) = p(b/w)$.

The resultant general expression for the loss of space A is as follows.

$$
\begin{aligned}
P_r(\text{loss of A}) = & \int_0^{w_{max}} dw \int_0^{h} dy \int_{x_1}^{x_2} dx \int_0^{b_{max}} db \left[p(b/w)p(x)\cdot p(y)p(w)\right] \\
& + \int_0^{w_{max}} dw \int_0^{h} dy \int_0^{x_1} dx \int_{2(x_1-x)}^{b_{max}} db \left[p(b/w)p(x)\cdot p(y)p(w)\right] \\
& + \int_0^{w_{max}} dw \int_0^{h} dy \int_{x_2}^{L} dx \int_{2(x-x_2)}^{b_{max}} db \left[p(b/w)p(x)\cdot p(y)p(w)\right]
\end{aligned}
\tag{5}
$$

Expressions for the probability of loss of space B or C or D can be found in a manner similar to that above. The only difference lies in the limits of the integrating

variables. Actually they can be found merely by substituting the correct x's in the integrals of Equation (5). For instance the probability of loss of space C would require substitution of x_3 for x_1 and x_4 for x_2 in Equation (5).

An example illustrating the use of Equation (5) is carried out in Appendix A.

2. Probability Density Functions

A few assumptions have been made which have enabled us to arrive at a general expression for the loss of any one designated space. It is felt that these assumptions have also clarified the problem considerably and have provided a basis for extension to other events. The question that arises now is what are the individual expressions for the various probability density functions. Unfortunately one cannot say exactly what form the functions take. Most matters of a probabilistic nature require extensive data to substantiate any claim as to the density function most suitable to describe the problem at hand.

Consider the case of finding a distribution to represent the probability that a ship is hit along any portion of its length. One must ask what type of weapon

is to be used or what method of attack or fire control is being used. Each facet, and there are many more, of the problem has an effect on the probability of hitting the ship. During World War II most of the attacks were conducted by means of optical sightings using the periscope. For this method it appears quite natural to sight in on the midships portion of the target in order to minimize the chances of a torpedo running ahead or aft of the ship. Some data has been accumulated which indicates that actual attacks were directed at the midships section. Appendix C shows a plot of the number of hits that occurred at various percentages of the lengths of the ships. The plot resembles a normal distribution and perhaps is the type of distribution one might intuitively expect. Statistical data of such a nature frequently follow a normal distribution as the amount of data or number of experiments increases. The normal distribution has a density function of the form

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad -\infty < x < \infty \tag{6}$$

where μ is the mean of the distribution, σ the standard deviation, and σ^2 the variance. For the case of torpedo firings at a ship, the distribution may have a mean at L/2

and a standard deviation of L/6. These representative figures are taken from the plot of Appendix C. The function is readily adaptable to any mean and standard deviation which experimental results may indicate.

We have mentioned the case for optical sightings and firings. This is rapidly becoming outmoded, if not already, by more advanced methods of attack which do not rely on visual sightings. Sonar has been improved to the point where attacks can be made solely from information obtained from sonar ranges and bearings. This method is further enhanced by the arrival of the acoustic homing torpedo which does not require the fire control accuracy demanded by the more conventional free running torpedo. The acoustic torpedo carries its own control unit and needs only the general area in which its target lies. Depending upon the nature of the acoustic torpedo different distributions for hitting the target may exist. First of all the acoustic torpedo utilizes the fact that ships of any sort generate noise. Propulsion and auxiliary machinery of all types transmit vibrations through their mountings to the hull and from there to the water which acts as the medium for transmitting the noise to the listener. This noise has a frequency spectrum which the acoustic torpedo

utilizes to complete its mission. By having its listening device tuned to a certain frequency the torpedo can home in on the source producing this frequency. Propellers have a frequency spectrum dominant in the low ranges of frequency whereas reduction gears and main propulsion machinery have spectra which peak in the high frequency range. It may be expected that these new weapons are highly accurate and possess a high probability of achieving a hit. Furthermore it seems logical to assume that the probability density function for hitting a target when utilizing the acoustic torpedo will peak in the region of the frequency source for which the torpedo is tuned. For the case of a torpedo tuned to frequencies in the range of those generated by reduction gears, the function may be broad and centered in the region of the main machinery spaces and near zero in the region external to these spaces. A simple density function satisfying these requirements is the uniform density function which is constant over a particular range and has a magnitude of the reciprocal of the range. An example of the uniform density function is shown in Figure II.

The density functions described above are those expected to be found when considering the distribution of hits in the x direction, along the length of the ship. These functions may extend to regions outside the boundaries

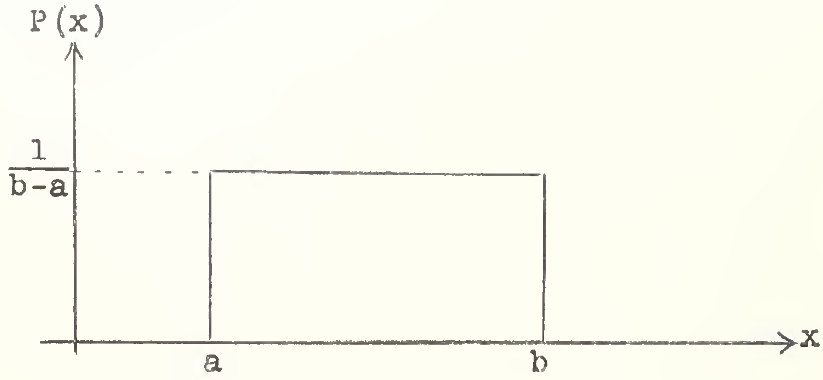


FIGURE II

Typical Uniform Density Function

of the ship, in which case they merely indicate a complete miss occurs. When considering the case for the distribution in the y direction, depth, then of course one must account for the fact that the distribution cannot extend into regions above the waterline. The function may extend below the keel of a ship, indicating either a miss or a possible underbottom explosion. It should be mentioned that this possible underbottom explosion may be more effective than the side contact hit but such a case will be considered in Part B. For the present we will assume that an attack falling outside the envelope of the ship is a miss and does not cause any damage to the side.

A function which can take the shape of a normal but has a lower limit of zero is the gamma density function

which has the form given in Equation (7) and shown in Figure III.[3]

$$P(y) = \frac{y^{\alpha} e^{-y/\beta}}{\beta^{\alpha+1}\Gamma(\alpha+1)} \qquad \begin{array}{r} 0 \leqslant y < \infty \\ \beta > 0 \\ \alpha > -1 \end{array} \tag{7}$$

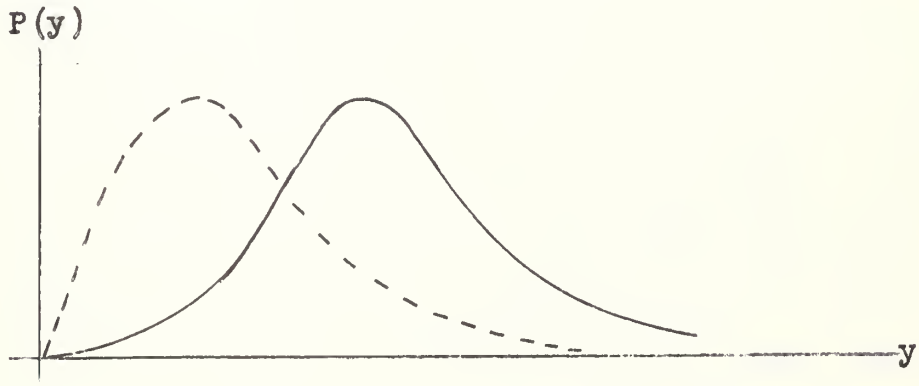


FIGURE III

Typical Gamma Density Function

The constants α and β are selected to shape the curve as desired. They act in a manner analagous to μ and σ in the more familiar normal distribution. The gamma function has a mean at $\beta(\alpha + 1)$. This mean could represent the depth at which a particular torpedo is meant to hit. If a torpedo can be expected to hit at one depth only, the density function would appear as a unit impulse at that particular depth. This of course could hold for the distribution in the x direction also.

The determination of p(w) is based mainly on intelligence data. We do not know exactly what the enemy

possesses as far as torpedo warheads are concerned. One can only estimate what the enemy possesses and what it may use based upon his own knowledge and experience in the field. It must be realized that there is an upper limit for the weight of a warhead that a torpedo can carry and still perform a job. We also realize that the enemy will not expend any effort on warheads of low weight which may not be capable of any significant damage. Just where these limits lie and where in the range there will be a concentration of effort is unknown and we can only postulate some distribution over an assumed range. For the present we will simply claim that p(w) can be represented by a uniform distribution over a range w_{min} to w_{max}. In an actual case one may have reliable information to indicate where these limits lie.

It remains now to determine what form p(b/w) may have. We are aware of the fact that the side protective systems are capable of defeating warheads below a certain weight, w_c. This critical weight is not definitely defined but is known to lie within a certain range as indicated by caisson tests. The side protection system is composed of many structural elements, the strengths of which can only be approximate especially after the various structures are worked into a system. So much workmanship is involved in constructing a system that

it is impossible to say just how perfect or imperfect the system actually is. Because of the numerous factors affecting the system one can only estimate its strength and do so conservatively. Below this assumed critical charge weight p(b/w) should be zero since no damage length is expected. Above this value the resultant damage length will increase in some proportion to the weight of charge used. If one was able to conduct a series of tests to determine these values of damage lengths as a function of charge weight, one might expect to obtain results similar to those shown in Figure IV.

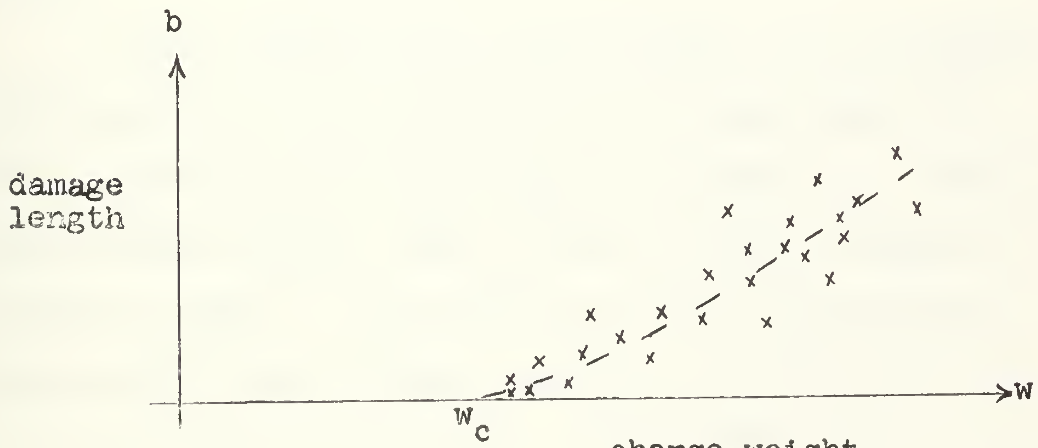


FIGURE IV

Representative Plot of Damage Length versus Charge Weight

For any specified charge weight used against a fixed side protection system the same damage length need not result from every test. Furthermore, the value of w_c may not be well defined. In general though one could assume that the average damage length $\bar{b}$ has the form of Equation (8)

$$\bar{b} = \begin{cases} 0 & \text{for} \quad w < w_c \\ k_o(w - w_c)^{\gamma_o} & \text{for} \quad w > w_c \end{cases} \tag{8}$$

where k_o and γ_o are constants which could enable one to fit the experimental data to the expression above.

We will assume that a large number, n, of caissons are available, all constructed in the same manner and each tested with the same weight of charge w_i. The results would indicate that a range of damage length b_i occurs n_i times. This data then in fact determines a probability that a damage length b_i results given that a charge weight w_i is used. If this data, that is, the percentages of time or the probability that the range b_i results given w_i is used, were plotted, a series of points would be obtained as shown in Figure V.

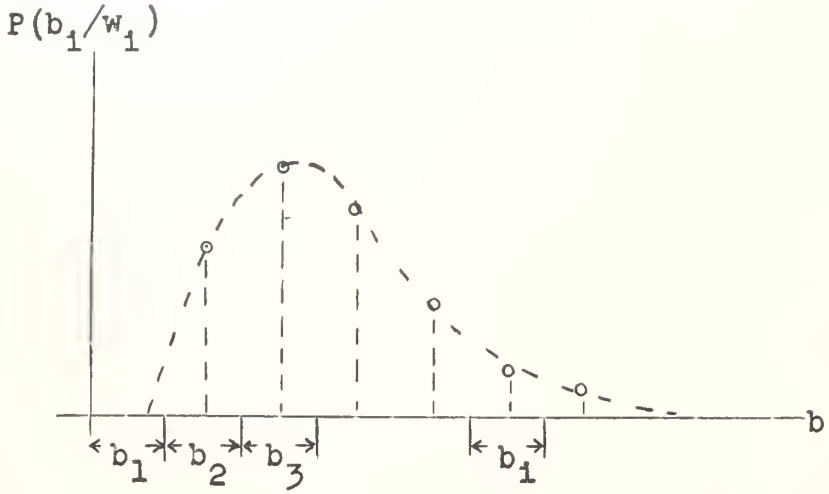


FIGURE V

Typical Plot of $p(b/w)$ for Discrete Case

In the above discussion a range of damage length b_i is mentioned. When plotting this we assume that the occurrence of a damage length in the range b_i corresponds to the occurrence of the mean of the range. If the number of caissons tested is permitted to become very large and the n_i computed for very small ranges b_i, then the plot approaches a continuous distribution which in the limiting case is essentially the probability density function for the occurrence of a damage length. If the same experiments were conducted for numerous w_i's then the family of curves representing $p(b/w)$ would result. A typical plot may appear as shown in Figure VI. There may be a minimum and certainly maximum damage length for any charge weight.

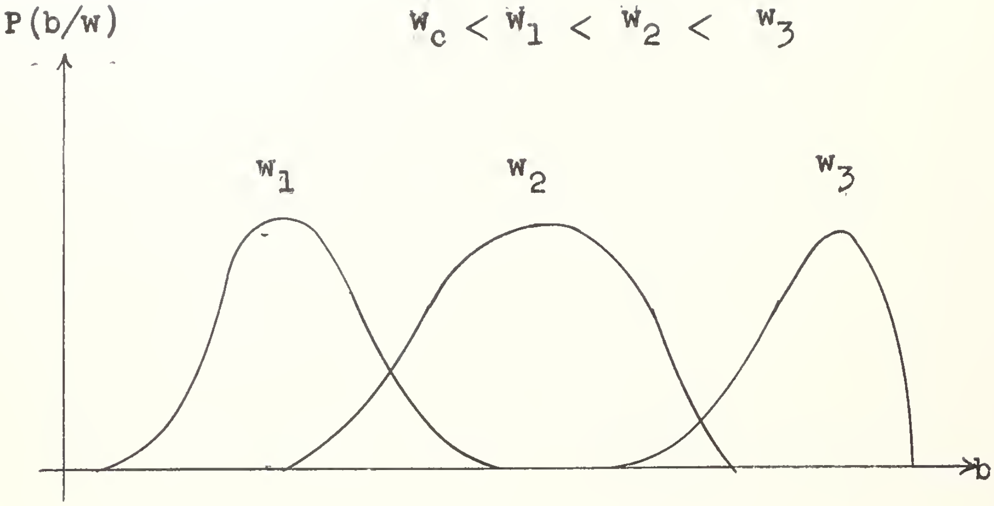


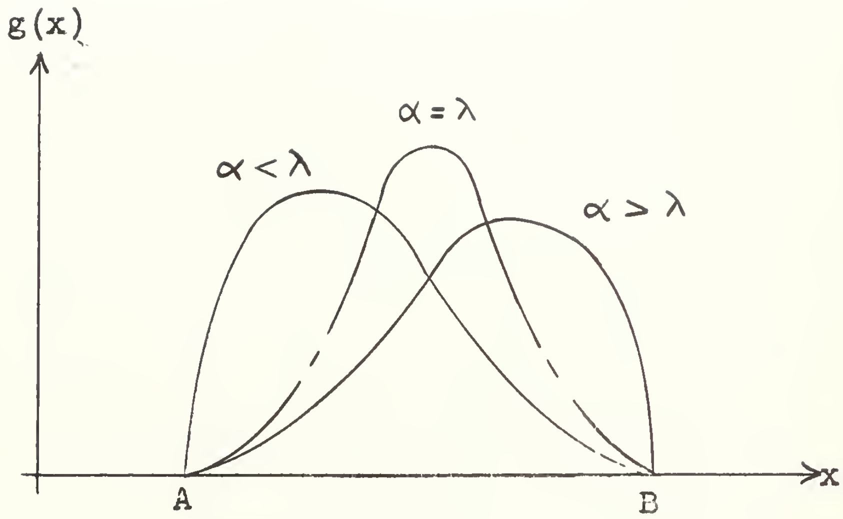
FIGURE VI

Typical Plot of p(b/w) for Continuous Case

For cases where an upper and lower limit exist, as expected in most cases, especially for higher charge weights, the beta density function is proposed.[4] The distribution permits arbitrary finite limits and shaping through the use of variable parameters. The form of the beta function is given in Equation (9) and plotted in Figure VII.

$$g(x) = \frac{1}{C}(x-A)^{\alpha} \cdot (B-x)^{\lambda} \qquad A \leqslant x \leqslant B$$

$$C = (B-A)^{\alpha+\lambda+1} \frac{\Gamma(\alpha+1)\Gamma(\lambda+1)}{\Gamma(\alpha+\lambda+2)} \tag{9}$$



<u>FIGURE VII</u>

Typical Plots of Beta Function

The parameters α and λ are arbitrary positive constants which are used to shape the curve in the region A to B. When applying this to the density function p(b/w), A and B could be made functions of w which would permit shifting the curve to higher values of b for larger values of w. It must be noted that for values of α and λ other than integers the expression is not explicitly integrable. It would not be difficult though to force α and λ to be integers close to what actual values the experimental data indicate should be used.

The use of the beta function as a probability density function for damage length is illustrated in example 4 of Appendix A.

It also might be feasible to approximate the experimental data by linear functions. For example the normal and beta function both lend themselves to triangular approximations. Since the triangle has finite limits this may suit the beta function quite well. For the case of the normal distribution one must insure that the tails of the normal which are lost include a negligible percentage of the total area.

The preceding discussion on density functions has been quite general and an attempt to predict and indicate what type distributions appear suitable for use in the various cases. Much of it has been based on intuition, mainly because of the lack of sufficient experimental data to provide substantiating proof. However, in each case simple methods of obtaining data which would provide the required information have been discussed.

So far a method of determining the probability of losing a single designated space has been presented. This usually is of minor significance as compared to the loss of any one of the four, or of any two spaces. The succeeding work will investigate the determination of such probabilities of loss.

3. Case of Loss of One Space

For the machinery arrangement shown in Figure I, the probability of losing one space is equal to the probability of losing A or B or C or D but exclusive of any combination of spaces. This is the case of exclusive events in which the overall event requires the occurrence of any one of its subordinate events and the non-occurrence of the remaining events.

The following notation will apply in the succeeding paragraphs.

A = Event "loss of space A"

A' = Event "Event A does not occur"

Similarly B, B', C, C', D and D' represent identical events pertaining to their respective spaces. With the above notation the following equality results.

$$P_r(\text{loss of one space, exclusive of more than one})$$
$$= P(AB'C'D') + P(A'BC'D') + P(A'B'CD') + P(A'B'C'D) \tag{10}$$

The event AB'C'D' occurs when a hit at x in the range 0 to x_1 results in a damage length greater than $2(x_1 - x)$ but less than $2(x_2 - x)$ to insure that event B does not

occur, or when a hit at x in the range x_1 to x_2 results in a damage length greater than zero but less than $2(x_2 - x)$. In integral form

$$P(AB'C'D') = \int_0^{w_{max}} dw \int_0^{h} dy \int_0^{x_1} dx \int_{2(x_1-x)}^{2(x_2-x)} db \quad P(x,y,b,w)$$

$$+ \int_0^{w_{max}} dw \int_0^{h} dy \int_{x_1}^{x_2} dx \int_0^{2(x_2-x)} db \quad P(x,y,b,w) \tag{11}$$

In order to refrain from writing out the actual integral in complete form we will let $P(x,y,b,w)$ equal $P(b/w)P(x)P(y)P(w)$.

By arguing limits for the remaining events in a manner similar to that for AB'C'D' the integral expression for these events follow.

$$P(A'BC'D') = \int_0^{w_{max}} dw \int_0^{h} dy \left[\int_{x_2}^{\frac{x_3+x_2}{2}} dx \int_0^{2(x-x_2)} db \quad P(x,y,b,w) + \int_{\frac{x_3+x_2}{2}}^{x_3} dx \int_0^{2(x_3-x)} db \quad P(x,y,b,w) \right] \tag{12}$$

$$P(A'B'CD') = \int_0^{w_{max}} dw \int_0^h dy \left[\int_{x_3}^{\frac{x_4+x_3}{2}} dx \int_0^{2(x-x_3)} db \quad P(x,y,b,w) \right.$$

$$\left. + \int_{\frac{x_4+x_3}{2}}^{x_4} dx \int_0^{2(x_4-x)} db \quad P(x,y,b,w) \right] \qquad (13)$$

$$P(A'B'C'D) = \int_0^{w_{max}} dw \int_0^h dy \left[\int_{x_4}^{x_5} dx \int_0^{2(x-x_4)} db \quad P(x,y,b,w) \right.$$

$$\left. + \int_{x_5}^{L} dx \int_{2(x-x_5)}^{2(x-x_4)} db \quad P(x,y,b,w) \right] \qquad (14)$$

The desired results, Equation (10) is the sum of the expression shown in Equations (11) through (14). Although the final result is quite lengthy its evaluation is simplified by the fact that many of the integrals are similar

with only the subscripts differing, so that evaluation of one leads to the evaluation of two or more of the others.

The application of Equations (11) through (14) is shown in example 2 of Appendix A.

4. Case of Loss of Two Spaces

The loss of just two spaces from a single hit requires that a hit occur near enough to the transverse bulkhead separating two spaces so that the resultant damage length extends into an adjacent space but not into a third space. For the side protective system in use and for bulkhead separation on the order of 40 to 50 feet it does not appear that damage length of this magnitude will occur from conventional charges, so that an extension of damage to a third space appears unrealistic. In actual cases the loss of more than two spaces from a single side contact hit is not considered feasible. However, in order to maintain the generality desired we must restrict the limits to exclude the third spaces. Figure I is duplicated below to indicate the regions of interest.

The probability of loss of two spaces, excluding more than two, is found utilizing the exclusive "or" as done in Section 3.

$$P(\text{loss of just two spaces}) = P(ABC'D') + P(A'BCD') + P(A'B'CD) \qquad (15)$$

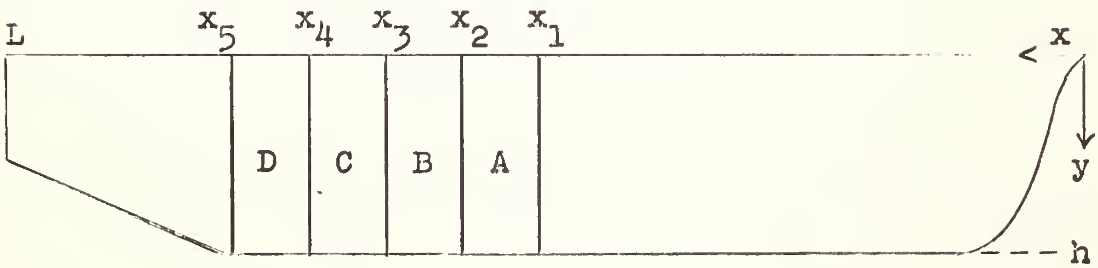


FIGURE I

Arrangement of Machinery Spaces

$$\begin{aligned} P(ABC'D') = {} & P_r(\text{hit at } x \text{ in range } 0 \text{ to } x_2) \cdot \\ & P_r(x_2 - x < b/2 < x_3 - x) \\ + {} & P_r\left(\text{hit at } x \text{ in range } x_2 \text{ to } \frac{x_3 + x_2}{2}\right) \cdot \\ & P_r(x - x_2 < b/2 < x_3 - x) \qquad (16) \end{aligned}$$

The second term on the right of Equation (16) places a restriction on x to be less than $\frac{x_3 + x_2}{2}$. This stems from the limits of b which require $x < \left[\frac{b}{2} + x_2\right]$ and $x < \left[x_3 - \frac{b}{2}\right]$. The only way for both of these inequalities to be satisfied is for $x < \frac{x_3 + x_2}{2}$.

$$P(ABC'D') = \int_0^{w_{max}} dw \int_0^{h} dy \left[\int_0^{x_2} dx \int_{2(x_2-x)}^{2(x_3-x)} db \quad P(x,y,b,w) \right.$$

$$\left. + \int_{x_2}^{\frac{x_3+x_2}{2}} dx \int_{2(x-x_2)}^{2(x_3-x)} db \quad P(x,y,b,w) \right] \qquad (17)$$

Similarly,

$$P(A'BCD') = \int_0^{w_{max}} dw \int_0^{h} dy \left[\int_{\frac{x_3+x_2}{2}}^{x_3} dx \int_{2(x_3-x)}^{2(x-x_2)} db \quad P(x,y,b,w) \right.$$

$$\left. + \int_{x_3}^{\frac{x_4+x_3}{2}} dx \int_{2(x-x_3)}^{2(x_4-x)} db \quad P(x,y,b,w) \right]$$

$$(18)$$

$$P(A'B'CD) = \int_{o}^{w_{max}} dw \int_{o}^{h} dy \left[\int_{\frac{x_4+x_3}{2}}^{x_4} dx \int_{2(x_4-x)}^{2(x-x_3)} db \quad P(x,y,b,w) \right.$$

$$\left. + \int_{x_5}^{L} dx \int_{2(x-x_4)}^{2(x-x_3)} db \quad P(x,y,b,w) \right] \tag{19}$$

P(loss of two space) = sum of Equations (17) to (19) (20)

In an actual case where the damage length could never exceed the length of the smallest machinery space, it would be impossible to lose more than two spaces from a single hit. Hence the probability of loss of two or more spaces would be given by Equation (20).

5. Addition of Auxiliary Spaces

It has been previously stated that auxiliary machinery spaces may exist between main machinery spaces, for the

purpose of reducing the probability of loss of more than one space from any particular hit. We will investigate now how such spaces effect the probabilities of loss of machinery spaces.

Consider two main spaces A and B separated by an auxiliary space F, as shown in Figure VIII.

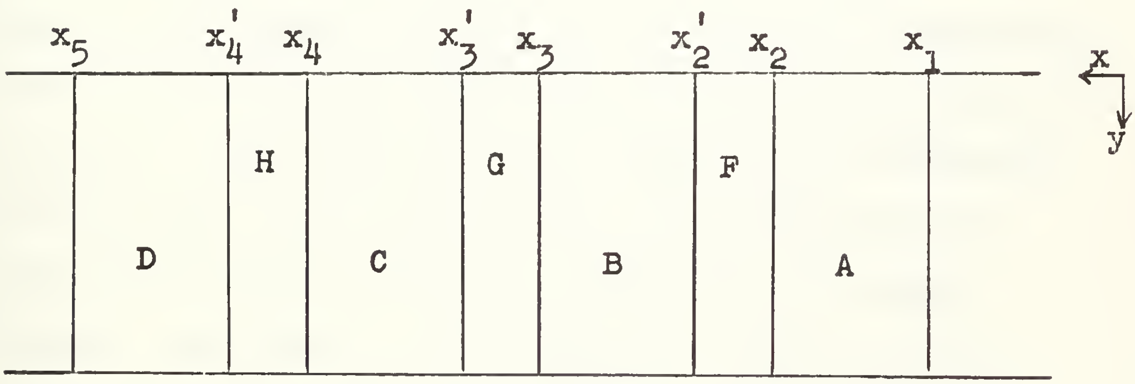


FIGURE VIII

Arrangement of Main and Auxiliary Machinery Spaces

For the case of a side contact hit we will assume that the loss of space F will not result in the loss of any shaft passing through the space. It is known that shock damage will result from side hits and there may

be some chance of shock causing misalignment of the shafting and supports. We will neglect this probability since it appears that it would be fairly small in ships possessing present protective systems.

The probabilities of loss of any of the spaces shown in Figure VIII can be calculated in the manner shown previously in Equation (5). The question that arises here is whether the addition of an auxiliary space actually reduces the various probabilities. First of all, assuming space D is as far aft as possible and G and H of Figure VIII are excluded, the addition of F results in an increase in the length of the machinery box and force A to move forward. The probability of hitting somewhere in the box is therefore increased. However, the addition of F separates A and B so that the chances of losing both A and B should be reduced. But because of the dependency of the result on the appropriate density functions one cannot say how much an effect this auxiliary space has on the overall problem. An illustrative example is carried out in Appendix A.

From our previous notation the loss of A and B exclusively is P(ABC'D'). With reference to Figure VIII

$$P(ABC'D') = P_r(\text{hit at } x \text{ in range } 0, x_2) \cdot P_r(x_2' - x < b/2 < x_3 - x)$$

$$+ P_r(\text{hit at } x \text{ in range } x_2, x_2') \cdot$$

$$P_r\left[\begin{matrix}\text{larger of} \\ (x_2' - x) \text{ or } (x - x_2)\end{matrix} < b/2 < x_3 - x\right]$$

$$+ P_r(\text{hit at } x \text{ in range } x_2' x_3) \cdot P(x - x_2 < b/2 < x_3 - x)$$

$$P(ABC'D') = \int_0^{w_{max}} dw \int_0^h dy \left[\int_0^{\frac{x_2'+x_2}{2}} dx \int_{2(x_2'-x)}^{2(x_3-x)} db \; P(x,y,b,w) \right.$$

$$\left. + \int_{\frac{x_2'+x_2}{2}}^{\frac{x_3+x_2}{2}} dx \int_{2(x-x_2)}^{2(x_3-x)} db \; P(x,y,b,w) \right] \tag{21}$$

It is easily seen that Equation (21) reduces to Equation (17) for $x_2' = x_2$ or for the removal of F. This is a general expression and does not bring out the obvious fact (although it is inherent in the limits) that the ex-

pression is zero unless the damage length is greater than $x_2' - x_2$. From the outset it might seem logical to make the distance between main machinery spaces greater than the maximum expected damage length. The difficulty here is that one may find the necessary separation may be quite large and one cannot afford the increase in length of machinery box and the extra length of shafting obviously made necessary.

The same approach may be taken for the addition of auxiliary spaces between B and C or between C and D, or for any combination of main and auxiliary spaces.

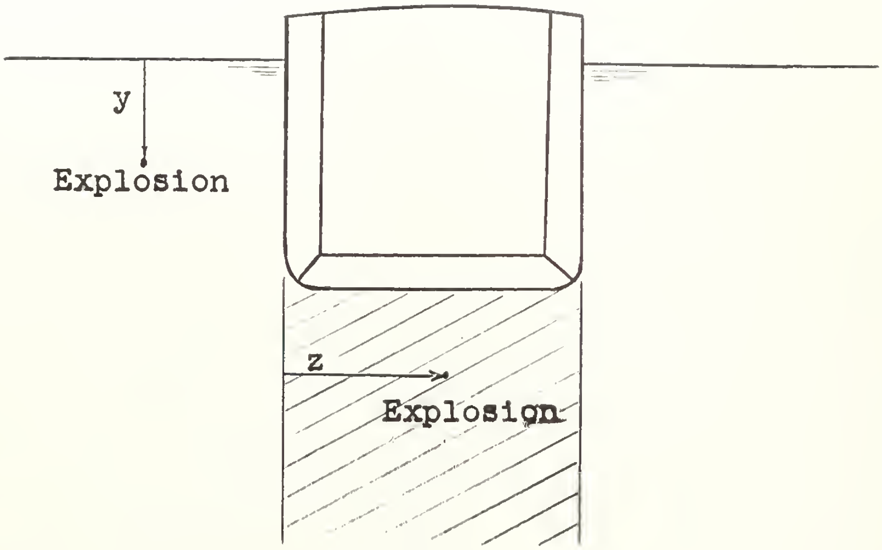
The integral expressions thus far determined can be useful in providing a format for the evaluation and comparison of different designs. Also, and more important, if the density functions lend themselves to explicit integration an optimization may be possible. For instance it would be useful to determine the best arrangement of auxiliary spaces with restriction on the total allowable length of auxiliary spaces.

An example illustrating the effect of the addition of an auxiliary space on the probability of loss of two or more spaces is presented in Appendix A.

B. Underbottom Non-Contact Explosion

In the design of a ship, the bottom protection system normally is smaller in size and hence less resistant to damage from an underwater explosion than the side protection system. If this were the only fact taken into consideration, it would appear that the case of the underbottom explosion would be just a variation of the non-contact side explosion case wherein the latter was rotated through 90 degrees in the vertical plane. Then with slight modifications, the results found previously could be applied directly to the underbottom case. One modification would be replacing the variable y, the distance from the surface of the water to the point of explosion, by the variable z, the horizontal distance under the ship from the side to the point of explosion. (See Figure IX)

Another modification would be that although the density function for $P(b/w)$ would have essentially the same shape, the location of the mean damage length would be to the right, i.e., $\bar{b}$ would increase for a similar charge weight at a fixed stand-off distance. This is shown in the figure below. (See Figure X)



Region of Underbottom
Attacks

FIGURE IX

Underwater Explosions

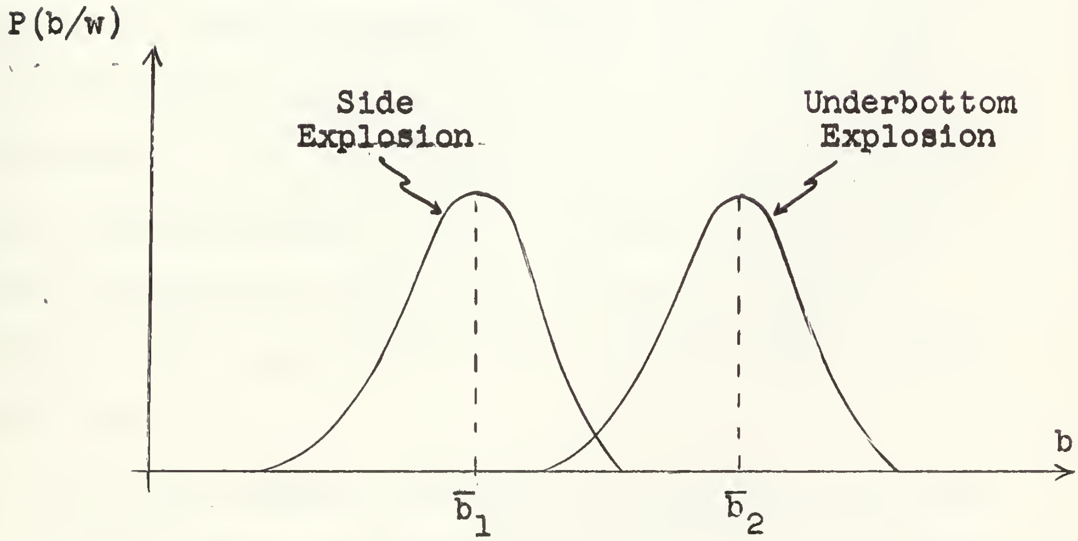


FIGURE X

Plot of P(b/w) for Side and Underbottom Non-Contact Explosions for Fixed Charge Weight and Stand-off Distance, Considering Shock-Wave Damage Only

However there is one big exception to this similarity between the side and underbottom explosions which cannot be overlooked and which will make the results obtained by the above variations inadequate. This exception is caused by the gas bubble which, due to its migration and pulsation, may contribute greatly to the extent and severity of damage if the explosion occurs under the ship. Hence the bottom damage will be the result of the shock-wave loading and the additional deformation due to bubble-pulse loading.

That there is sufficient energy in the bubble-pulse loading to cause substantial damage is shown by the energy balance diagram in Figure XI, which disregards gravity migration.(2) It can be seen that the energy in the first pulsation is almost half the total energy released by the explosion, whereas the energy radiated as first bubble pulse is equal to slightly more than one-third the damage energy in the shock wave.

At first glance, the above discussion would seem to indicate that the underbottom explosion case could still be solved by using the previously mentioned variations of the results of the side explosion case

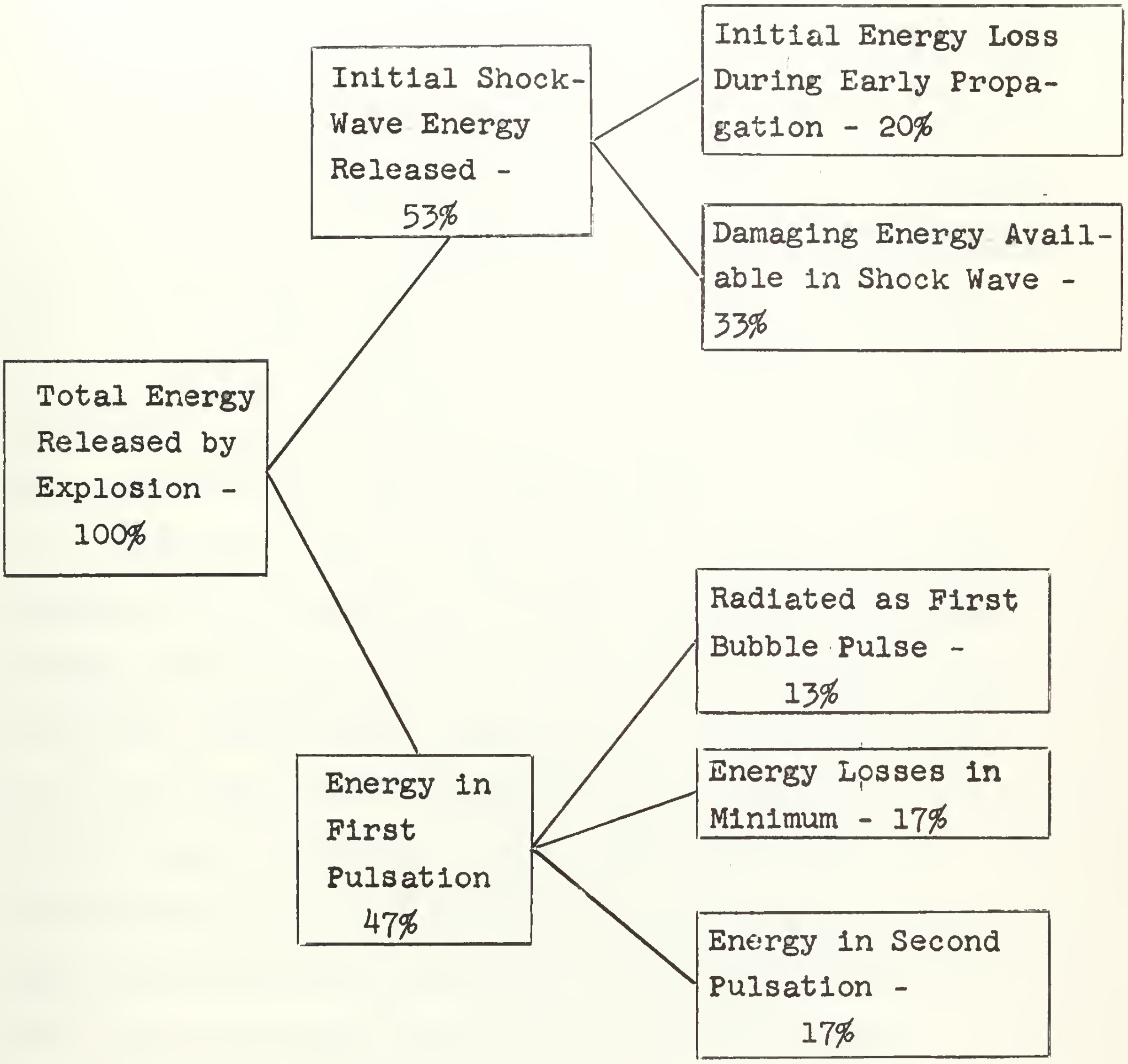


FIGURE XI

Energy Balance Diagram for an Underwater Explosion

and including a factor to compensate for the bubble-pulse loading. For example, assuming a linear increase in damage length with respect to damaging energy, then a simple shift of $\overline{b}_2$ to $4/3\ \overline{b}_2$ for the density function of Figure X should include the additional damage effect due to the energy radiated in the first bubble pulse. However further consideration will show that this solution is not feasible since the underbottom explosion is not just a simple case of added energy.

First the longitudinal extent of damage would be greater in the underbottom explosion case than in the simple added energy case. This results from the fact that the bubble-pulse loading occurs at a later time than the shock-wave loading because of the time required for a bubble pulsation. Since the ship is assumed to be moving, the bubble-pulse loading will generally be directed against a different portion of the bottom structure than the shock-wave loading. Therefore the distance between the two locations will depend on the pulsation time of the gas bubble and the speed of the ship. Hence these two variables must be included in any results obtained.

Another factor that must be considered is that when the gas bubble is pulsating, it is also migrating

upward toward the ship. Hence one of the bubble pulses may occur near the ship's bottom, causing a very high and very localized loading to develop, and subjecting the bottom structure to excessive load. This loading is aided by the superposition of the hydrodynamic mechanism of bubble collapse. The bubbles of large explosions, such as occur with mines or torpedoes, lose their symmetry during the first oscillations due to the difference of hydrostatic pressure between the top and the bottom of the bubble. During the contraction of the bubble toward its minimum, the water near the bottom of the bubble moves much faster toward the center than the water on the sides, which itself is moving faster than the water at the top. This leads to a huge, high-velocity water jet penetrating the bubble and forming a torus.[5] This water jet, along with the high-pressure field surrounding the bubble at that time, is extremely efficient in producing damage. This mechanism of bubble collapse is shown in Figure XII.[2]

Thus bubble-pulse loading is an important consideration since it can lead to a rupture or a hole in the bottom protection system in several ways. If the first bubble pulse occurs close enough to the bottom, the intense localized loading could cause a hole in the bottom

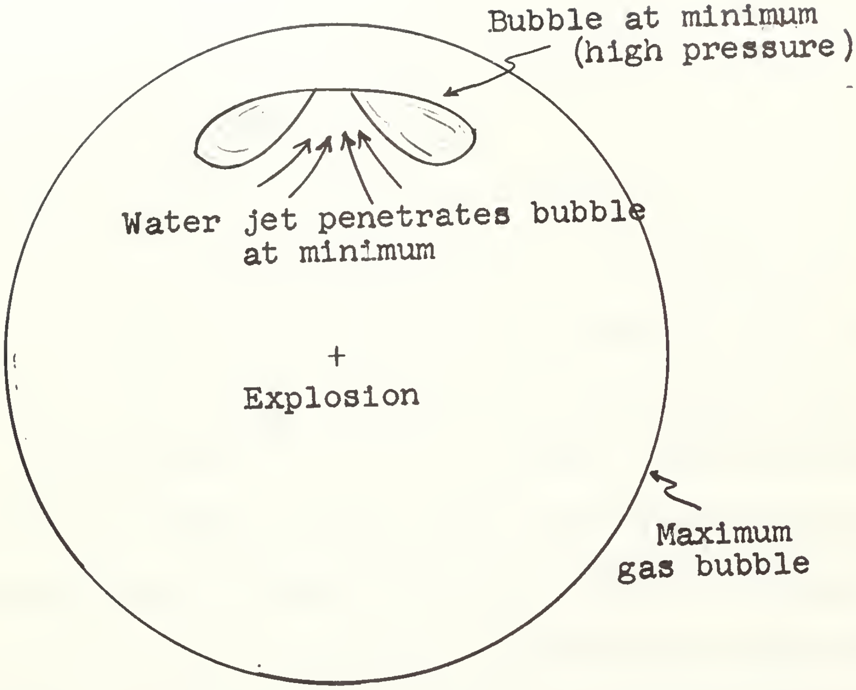


FIGURE XII

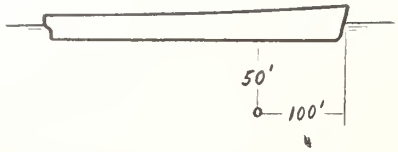
Mechanism of Bubble Collapse

structure, with the subsequent release of the remaining bubble energy into the attacked compartment, and possibly leading to the loss of a shaft passing through the compartment. For a sufficiently strong bottom or a not too severe bubble-pulse loading, the bottom could be deformed or brought to rupture at the restraints offered by the bulkheads. Since any deformation of the bottom is connected with a severe load transmission into the bulkheads, then buckling of the bulkhead plating can occur,

along with deformation of the bulkhead stiffeners, which could lead to tearing and loss of watertight integrity.[2]

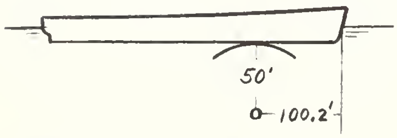
It is this type of damage that will be considered, although other damage could be as important but would be difficult to include in this analysis. For example as opposed to a contact explosion, when a mine or some sort of influence fuzed weapon explodes at a distance under the hull, there normally will be a wider distribution of equipment shock damage coupled with a general reduction of the longitudinal strength which can result in greater jeopardy for the ship. Further complications would arise if an attempt were made to include in the analysis the effects from the whipping or flexural vibration of the ship as a flexible beam. This occurs because the initial shock wave will start the ship vibrating, with the response depending on the location of the attack with respect to the nodal points of the various modes of flexural vibration. In addition the sequence of bubble pulses may tend to reinforce or reduce the amplitude of a particular mode. The amount of this effect of the pulses will vary with the degree of damping, the underpressures associated with bubble expansion, and the type of failures occurring in the hull structure during the first phases of the overall whipping process.[1]

A good example of the sequence which could occur with an underbottom non-contact explosion is shown in Figure XIII.[1] Several important facts can be seen from this sequence. Since the time between detonation and emission of the shock wave to its arrival at the bottom structure of the ship is extremely small and the migration or vertical velocity of the gas bubble is also small, then the vertical change in the location of the gas bubble can be neglected. During the next phase, the expansion of the gas bubble to its first maximum, the time interval is large, so that even though the bubble velocity is small, the vertical change in bubble location cannot be neglected. When the bubble is contracting to its minimum size, its vertical velocity is much greater and there is a noticeable change in its vertical location. These same changes are repeated during the next sequence of time intervals; emission of pressure pulse to arrival of the first bubble pulse at the ship's bottom, expansion to second bubble maximum, and contraction to second minimum.[6] This variation of bubble size and migration with time, along with the corresponding pressure changes, are shown in Figure XIV.[2] During all these phases, the velocity of the ship can be

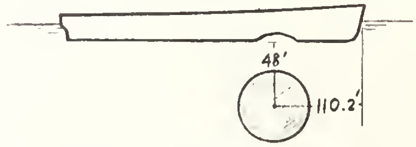


(1) Time - 0 milliseconds
Detonation
Emission of Shock Wave

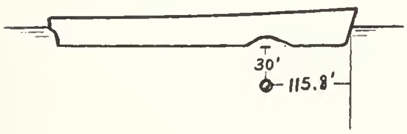
640 lb. chemical charge
50 ft. under keel
100 ft. from bow
waterline



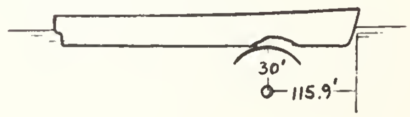
(2) Time - 10 ms
Arrival of Shock Wave



(3) Time - 600 ms
First bubble maximum

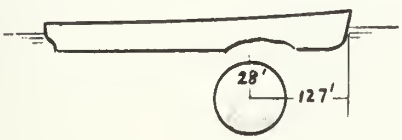


(4) Time - 930 ms
First Bubble Minimum
Emission of Pressure
Pulse

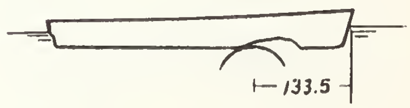


(5) Time - 936 ms

Arrival of First
Bubble Pulse



(6) Time - 1600 ms
Second Bubble Maximum



(7) Time - 1940 ms
Collpase of Second
Bubble Against Hull
Accompanying
Pressure Pulse

FIGURE XIII

Underbottom Non-Contact Explosion Sequence

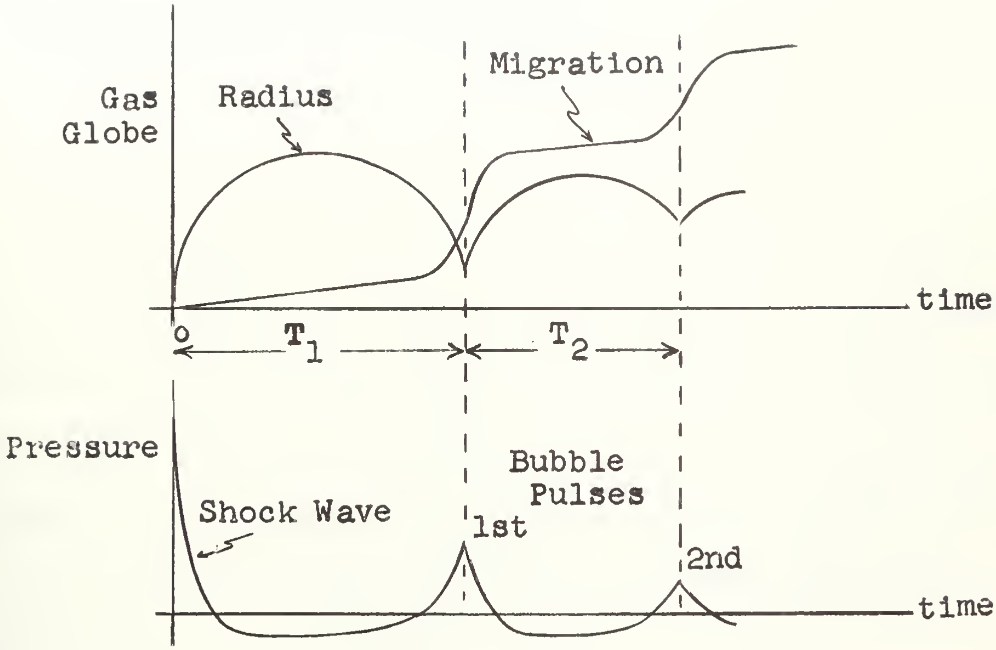


FIGURE XIV

History of Underwater Explosion Events (Schematic)

assumed to be constant, so that the changes in the horizontal location of the gas bubble will be linearly proportional to the time intervals between phases.

1. Determination of Probability Density Functions

For the first example, consider the influenced fuzed torpedo aimed to pass under the ship's bottom before exploding. The location of the point of explosion along the coordinate axes of the system consists of random variables assumed to be independent of each other. The probability

of the explosion occurring along the length of the ship, P(x), will be the same as for the side attack, since the only difference in the two attacks is the depth at which the torpedo is set to run. If it is assumed that the attack is carried out optically, this probability will be a normal distribution as stated in Appendix C, with the mean being the point of aim. Using sonar and acoustic torpedoes, of course, the distribution curve would peak at that location in the ship which contains the frequencies set in the weapon.

If one were to consider the case of a mine instead of the torpedo, the above density function would be slightly different. For P(x), the same normal distribution again could be expected for the longitudinal location of the point of explosion, but its mean generally would be shifted to that point of the ship at which the mine is set to explode. For the other density functions to be determined, it can be expected that it will be immaterial whether the weapon is a torpedo or a mine.

It seems logical to assume that the torpedo or the mine will be fired or laid at a depth such that the weapon explodes at a distance under the ship so as to obtain the maximum effective damage from the shock-wave

loading and the bubble-pulse loading. Therefore this depth could be a function of the charge weight, since the larger the weight the greater the depth can be for a specific damage. In addition, this depth will of necessity be a function of the draft of the ship if this proper distance for explosion under the ship is to be obtained. Since we are considering only the case of aircraft carriers, we can assume a fixed draft and hence remove this dependency. Therefore we shall have a probability of depth given charge weight, P(y/w). For any particular charge weight, and over a large number of firings, it can be expected that the points of explosion will be normally distributed in depth, with the mean value being the optimum depth for this weight.

In determining the number of spaces that could be lost, the important criterion is the longitudinal extent of the length of damage, b. In all cases, this damage length will be assumed to be longitudinally symmetrical about the point of explosion. Therefore it is important to consider the amount of damage that could be expected from the shock-wave loading and the subsequent bubble-pulse loadings. Here again assumptions can be made in an attempt to simplify the problem enough to obtain some meaningful results.

In the section pertaining to the side contact explosion, some arguments were given for the probability density function $P(b/w)$, that is, the probability that the damage length is in a particular range given that the charge weight was in a certain range. This will of necessity be a function of the design of the protection system. Now for the case of underbottom noncontact explosions, we find that the damage length is a function of not only charge weight, but of the depth of the explosion below the ship--the stand-off distance. This is intuitive and certainly appears realistic. For a fixed charge weight, the damage will definitely change as the depth of the explosion is decreased or increased. Just how this change occurs and what its magnitude actually is can only be accurately determined by numerous experiments. However, some understanding of the mechanisms of the underbottom explosion can lead to an approximation to the form of the density function $P(b/w,y)$.

If one considers the initial shock wave as the sole source of damage, the matters are somewhat simplified. For a fixed charge weight we would expect the greatest damage length or area to occur for shallow depth of explosion, i.e. for y small. As the depth of explosion increases, the shock wave must travel a greater distance

and there would be a decrease in energy per unit area of the wave front. Also the curvature of the wave front decreases so that the wave front contacting the ship covers a greater portion of the length. (See Figure XV) This energy is spread over this area in a lesser concentration. Hence although the total energy absorbed may be of the same order of magnitude, it is more uniformly

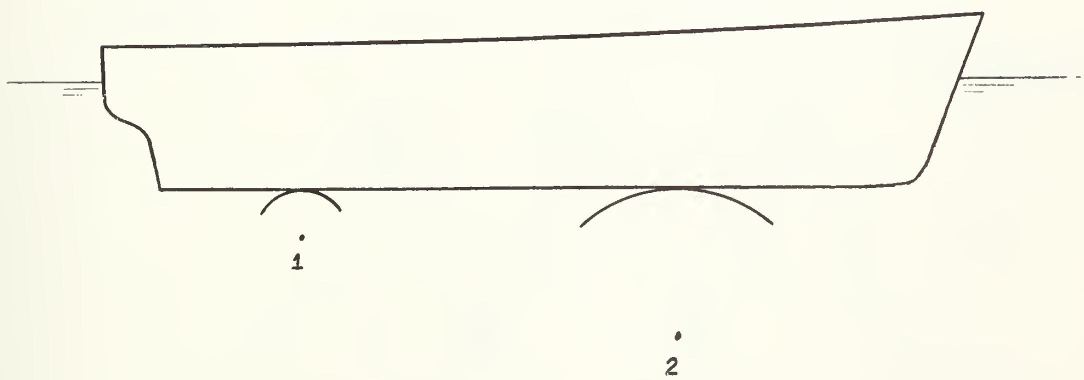


FIGURE XV

Arrival of Shock Wave

spread out over the contact area. At some depth of the explosion, this energy arriving at the ship would not be sufficiently concentrated to create a rupture in the ship's bottom, although it may create enough shock damage to be

equally effective. However it is difficult to evaluate this shock damage in probabilistic terms. Nevertheless, for the fixed charge weight as the depth of the explosion increases, the damage length is expected to decrease. A representative density function for such a case might appear as shown below in Figure XVI. There is little cause to expect that it would be greatly different in form from that of the side contact explosion.

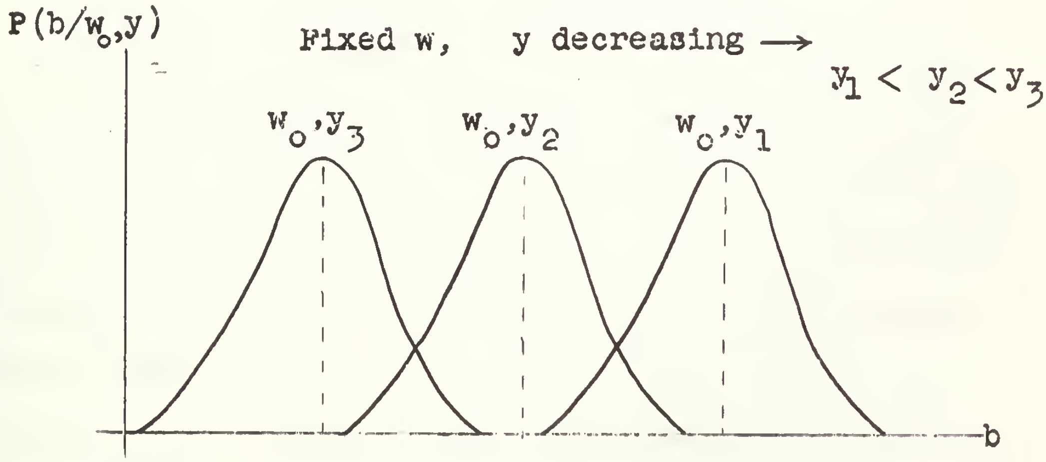


FIGURE XVI

Probability Density Function $P(b/w_o,y)$

Similarly, for the case of the fixed y and varying w, we again visualize the same type of density function, with the mean damage length increasing with increasing w. Thus

a beta distribution might appear feasible for representing $P(b/w,y)$.

$$P(b/w,y) = \frac{1}{C}(b-A)^{\alpha} \cdot (B-b)^{\lambda} \tag{22}$$

$$A < b < B$$

α, λ are constants

$$C = (B-A)^{\alpha+\lambda+1} \quad \frac{\Gamma(\alpha+1)\,\Gamma(\lambda+1)}{\Gamma(\alpha+\lambda+2)}$$

For this case A and B could be made functions of w and y so that the mean of the distribution shifts back and forth as w and y vary. For instance a function $k_1 \frac{w}{y}$ has this type of behavior in that at fixed y, increasing w increases the function, while at fixed w, decreasing y increases the function. This is the type of behavior mentioned above. Actual experiments would be needed to determine if the actual function should be $k_1 \frac{w^a}{y^b}$, where k_1, a, and b are constants.

The energy diagram of Figure XI shows that the largest amount of energy is concentrated in the initial shock wave, but that the first bubble pulse contributes an amount in the order of 13 percent. If this energy is expended in a region already weakened by the shock wave, then there is a

possibility that this fixed pulse could create the rupture should the shock wave fail to do so. It may even be possible for the first pulse to create a second rupture following that due to the shock wave if certain conditions are met. These conditions are functions of energy of the charge, depth of explosion, and speed of the ship. Consider the example shown in Figure XIII of an explosion of charge weight 640 pounds occurring at a depth of 50 feet below the keel. There is a delay of 926 milliseconds between arrivals of the shock wave and first bubble pulse. If we assume that the speed of the ship is 30 knots, then during these 926 milliseconds the ship could have moved a distance of 50 feet, so that the points of contact of the respective shock waves might occur 50 feet apart. If each were capable of rupturing the bottom, then one is faced with the problem of two hits. At this distance apart, they could easily result in the loss of two and possibly even three machinery spaces, provided that the energies were sufficient to do a reasonable amount of damage. Just how realistic this is could be a matter for discussion. It might require a considerable amount of charge before 13% of its energy would reach a value sufficient to create a rupture in the bottom. By then the original shock wave energy probably would have been enough to create considerable damage so that the first pulse appears insignificant.

Because of the bubble-pulse loading, one may be faced with a damage length, possibly discontinuous in nature, which is dependent upon factors such as the speed of the ship, the depth of the explosion, and the charge weight. An approach to the problem might be to consider the one explosion as two with a separation in the x-direction of magnitude $V_s \tau$, where V_s is the speed of the ship and τ is the time interval between arrival of the shock wave and the first bubble pulse. With this model it might be possible to arrive at an expression for particular losses given that two hits occurred, and then to weight these by the probability that such a case occurs. We do not intend to investigate this problem further since it would be another order of magnitude more difficult than the single hit.

Another probability density function that must be taken into account is that relating to the horizontal location of the point of explosion from the side of the ship, P(z). For the case of the mine, it can be expected that the explosion is equally likely to occur anywhere under the ship in the range of the beam. Hence a uniform probability density function can be assumed. For a fuzed torpedo, a normal distribution can be assumed for the location of the explosion in the z-direction, with the

mean being at the centerline of the ship. This brings a third dimension into the calculations as shown below in Figure XVII.

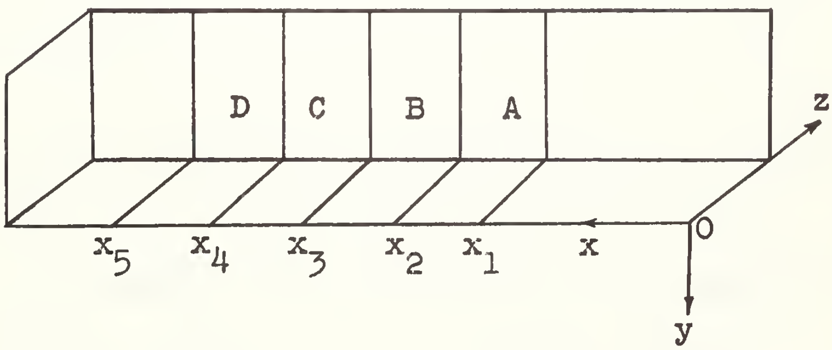


FIGURE XVII

Three Dimensions Used for the Underbottom, Non-Contact Explosion

2. Case of Loss of a Particular Space

Let us now consider the case of determining the probability of loss of a machinery space as a result of an underbottom, non-contact explosion. For the present we shall assume that there are no intervening spaces between the main machinery rooms, these latter being designated A, B, C, and D. With reference to Figure XVII, one of the collowing conditions must be met in order to cause loss of space A:

1. An explosion occurs below space A at a point x within the range x_1 to x_2, at z within the range 0 to B_m, and at y in the range 0 to y_{max}, where y_{max} is the greatest depth at which the explosion can occur and still cause a rupture in the bottom protection system for the charge weight used;

2. An explosion occurs at x in the range 0 to x_1 and that one half of the damage length is greater than $(x_1 - x)$, at z between 0 to B_m, and at y in a range so as to satisfy the condition on the damage length;

3. An explosion occurs at x in the range x_2 to L and that one half of the damage length is greater than $(x - x_2)$, with the same requirements for z and y as given for case 2.

As stated above, the fulfillment of any of these three conditions will result in the loss of space A. Since the conditions are mutually exclusive events due to the limitations on x, the total probability of the loss of space A will just be the sum of the probabilities of each event occurring. The evaluation of this will proceed in the same manner as for the side case. There-

fore, the probability of loss of space, P(A), is as follows.

$$P(A) = \int_0^{w_{max}} dw \int_0^{B_m} dz \int_0^{y_{max}} dy \int_{x_1}^{x_2} dx \int_0^{b_{max}} db \left[P(b/w,y) \cdot P(x)P(y) \cdot P(z)P(w) \right]$$

$$+ \int_0^{w_{max}} dw \int_0^{B_m} dz \int_0^{y_{max}} dy \int_0^{x_1} dx \int_{2(x_1-x)}^{b_{max}} db \left[P(b/w,y) \cdot P(x)P(y) \cdot P(z)P(w) \right]$$

$$+ \int_0^{w_{max}} dw \int_0^{B_m} dz \int_0^{y_{max}} dy \int_{x_2}^{L} dx \int_{2(x-x_2)}^{b_{max}} db \left[P(b/w,y) \cdot P(x)P(y) \cdot P(z)P(w) \right] \qquad (23)$$

It can be seen that this expression is similar to that developed for the side contact hit with the difference being that the damage length in this case has a distri-

bution dependent upon depth as well as charge weight and that there is a third ddmension. The probabilities of loss of B, C, and D can be readily determined merely by altering the limits on the above integrals for P(A). For instance P(B) is found by substituting x_2 for x_1 and x_3 for x_2 in the expression for P(A). An example of the use of Equation (23) is shown in Appendix B.

Various other probabilities could be determined, for example the loss of just one or two spaces. This has already been done for the case of the side contact hit. The development of these probabilities for the underbottom non-contact case would follow the same pattern with the only difference being that mentioned in the above paragraph. Therefore these expressions will not be developed.

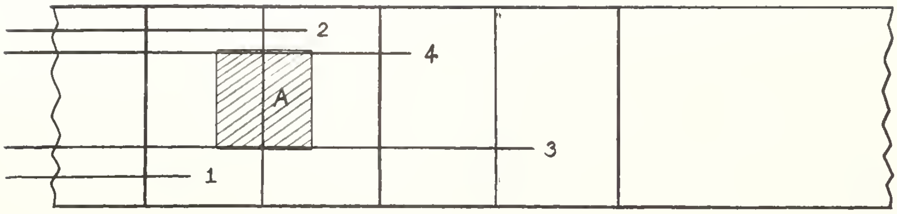
3. Case of Loss of a Particular Shaft

So far we have discussed only the probability of the loss of machinery spaces as a result of an underbottom explosion. Of equal importance is the probability of the loss of shafts. This presented no problem in the side contact case since, as was stated before, if a space was lost, then the shaft which started in that space was

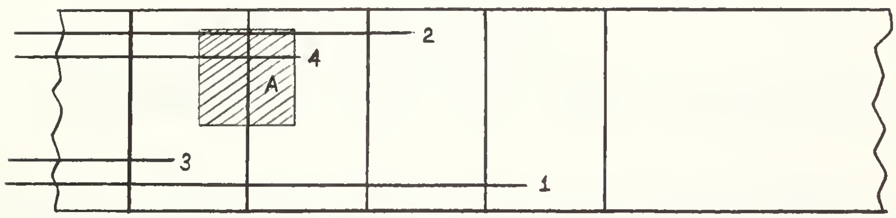
also considered lost. Therefore the various probabilities regarding loss of shafts equaled the corresponding probabilities of loss of spaces.

However this fact will not be true for the underbottom case. Of course we shall still assume that if the explosion defeats the protective system and the space is lost, then the shaft originating in that space is also lost. But additional shafts could be lost even though they start in a space remote from the location of the explosion. This can happen because the shaft could extend through the damaged space with the explosion occurring in close proximity to it. Then the damage would either physically destroy the shaft or else deform the bottom plating on which the shaft is supported to such an extent that the shaft is misaligned. In either case the shaft is rendered useless and hence is lost.

In connection with this, Figure XVIII shows how the shaft arrangement can be varied so as to obtain the best possibility of smallest maximum loss of shafts from an underbottom hit. With Arrangement I, a damage area A will cause loss of shafts 1 and 2 through loss of space and shafts 3 and 4 through damage, thus giving a maximum possible loss of four shafts. With Arrangement II, the same damage area can give only a maximum possible loss of



ARRANGEMENT I



ARRANGEMENT II

FIGURE XVIII

Arrangement of Shafts to Minimize Loss From An Underbottom Hit

three shafts. Hence from this consideration, of the two arrangements shown out of the many that could have been, Arrangement II would be the better one.

Consider the arrangement of machinery rooms shown below in Figure XIX. As just mentioned, the shafting could have a number of possible arrangements. In order to develop a general expression for the probability of loss of any particular shaft we shall consider the case of that shaft originating in machinery space A.

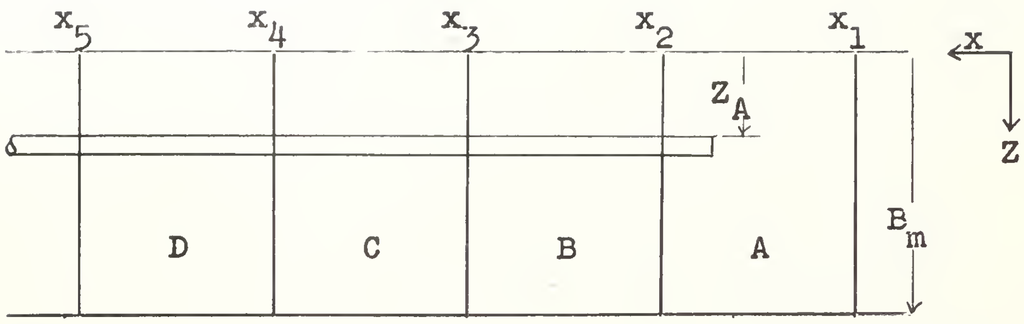


FIGURE XIX

Arrangement of Machinery Rooms and Shaft A

Shaft A (although not necessarily just shaft A alone) can be lost by either of two ways; first, by the loss of the space in which it originates, and second, as a result of damage to any portion of its length extending to its termination at the propellor. This secondary damage for the case of shaft A could result form an explosion under spaces B, C, D, or further aft, which might rupture the platform upon which one of shaft bearings is supported, or by the indirect effect of a misalignment caused by the fact that a rupture will deform the platform in the region around the shafting. This brings up the fact that the effective damaging area could be greater than the area of actual rupture. A modification to the density function P(b/w,y) would be in order. It might be possible to simply scale b to include the effect of deformed but in-

tact plating under the line bearings. Therefore one might term the damage length b to be equal to kb_r, where k is some scale factor and b_r is the length of the rupture.

The probability of loss of shaft A from one under-bottom explosion is therefore equal to the probability of occurrence of either or both of the two aforementioned events and will be designated by P(SA). As developed in the preceding section, P(A) is the probability of loss of space A. If we denote the probability of loss of shaft A due to a hit along its length external to A by $P(T_A)$, then

$$P(SA) = P(A \text{ or } T_A) = P(A) + P(T_A) - P(AT_A) \qquad (24)$$

The combined event AT_A can occur when a hit occurs aft of space A in the region about shaft A with sufficient strength to create damage extending into space A. $P(T_A)$ is then the probability that a hit occurs at x in the range x_2 to L_A' (the point at which shaft A penetrates the hull), at z in the range 0 to B_m, at y in the range 0 to y_{max}, and that the half damage or deforming length, b/2, is greater than the magnitude of $z-z_A$, where z_A is the position of shaft A in the z-direction as shown in Figure XIX. Therefore we can write

$$P(T_A) = \int_0^{w_{max}} dw \int_0^{B_m} dz \int_0^{y_{max}} dy \int_{x_2}^{L_A'} dx \int_{2|z-z_A|}^{b_{max}} db \left[P(b/w,y) \cdot P(x)P(y) \cdot P(z)P(w)\right] \tag{25}$$

The magnitude sign in the above integral would prevent a direct evaluation. However the integral could be divided into two cases, $z > z_A$ and $z < z_A$. The limits on z would therefore be effected for each case. Taking these factors into account, the expression for $P(T_A)$ is as follows:

$$P(T_A) = \int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_2}^{L_A'} dx \int_0^{z_A} dz \int_{2(z_A-z)}^{b_{max}} db \left[P(b/w,y) \cdot P(x)P(y) \cdot P(z)P(w)\right]$$

$$+ \int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_2}^{L_A'} dx \int_{z_A}^{B_w} dz \int_{2(z-z_A)}^{b_{max}} db \left[P(b/w,y) \cdot P(x)P(y) \cdot P(z)P(w)\right] \tag{26}$$

$P(AT_A)$ is the probability that a hit occurs at x,y, and z as before except that now half the damage length, b/2, must be greater than $|z - z_A|$ or $(x - x_2)$, whichever of these two is larger. The region for the probable occurrence of event AT_A is shown in Figure XX as the cross-hatched or lined area to the left of x_2.

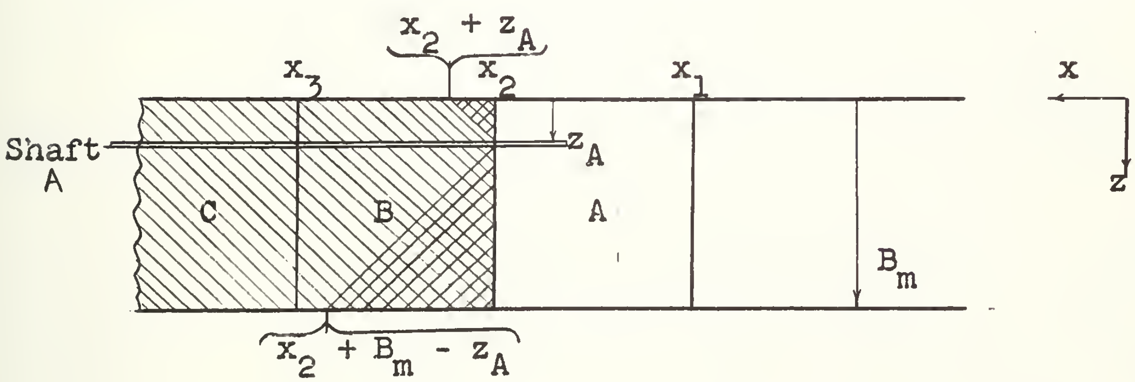


FIGURE XX

Region of Probable Occurrence of Event AT_A

The event AT_A occurs for any hit in the cross-hatched region of Figure XX provided that the half damage length is greater than $z_A - z$ for $z < z_A$ or greater than $z - z_A$ for $z > z_A$. This insures that the damage will extend into space A as well as to the shaft A in the region external to space A. This also occurs for the case of the lined area when $(x - x_2) > |z - z_A|$.

Let

$$P(i) = P(b/w,y)P(z)P(x)P(y)P(w) \qquad (27)$$

Then the integral expression for $P(AT_A)$ is

$$P(AT_A) = \int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_2}^{x_2+z_A} dx \int_0^{z_A-(x-x_2)} dz \int_{2(z_A-z)}^{b_{max}} db \quad P(i)$$

$$+ \int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_2}^{x_2+(B_m-z_A)} dx \int_{z_A+(x-x_2)}^{B_m} dz \int_{2(z-z_A)}^{b_{max}} db \; P(i)$$

$$+ \int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_2}^{x_2+z_A} dx \int_{z_A-(x-x_2)}^{z_A+(x-x_2)} dz \int_{2(x-x_2)}^{b_{max}} db \quad P(i)$$

$$+ \int_{o}^{w_{max}} dw \int_{o}^{y_{max}} dy \int_{x_2+z_A}^{x_2+B_m-z_A} dx \int_{o}^{z_A+(x-x_2)} dz \int_{2(x-x_2)}^{b_{max}} db \quad P(i)$$

$$+ \int_{o}^{w_{max}} dw \int_{o}^{y_{max}} dy \int_{x_2+B_m-z_A}^{L_A'} dx \int_{o}^{B_m} dz \int_{2(x-x_2)}^{b_{max}} db \quad P(i)$$

(28)

Substituting Equations (23), (26), and (28) into Equation (24) will give the integral expression for the probability of loss of shaft A.

The probability of loss of shaft B can be determined in a manner similar to that done for shaft A. In this case we shall denote the various analagous probabilities by P(SB), P(B), $P(T_B)$ and $P(BT_B)$. Thus

$$P(SB) = P(B) + P(T_B) - P(BT_B) \qquad (29)$$

By using Equation (23) and the information contained in the paragraph following this equation, as well as

utilizing Equation (27), the probability of the loss of space B can be determined.

$$P(B) = \int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_0^{B_m} dz \int_{x_2}^{x_3} dx \int_0^{b_{max}} db \quad P(i)$$

$$+ \int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_0^{B_m} dz \int_0^{x_2} dx \int_{2(x_2-x)}^{b_{max}} db \quad P(i)$$

$$+ \int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_0^{B_m} dz \int_{x_3}^{L} dx \int_{2(x-x_3)}^{b_{max}} db \quad P(i) \qquad (30)$$

$P(T_B)$ is the probability of losing shaft B from hits occurring in the area aft of space B. This is most easily determined by noting that the conditions are the same as that for $P(T_A)$ with the substitution of x_3 for x_2, L_B' for L_A', and z_B for z_A.

$$P(T_B) = \int_0^{w_{max}} dw \int_0^{y_{max}} dz \int_{x_3}^{L_B'} dx \int_0^{z_B} dz \int_{2(z_B-z)}^{b_{max}} db \quad P(i)$$

$$+ \int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_3}^{L_B'} dx \int_{z_B}^{B_m} dz \int_{2(z-z_B)}^{b_{max}} db \quad P(i) \qquad (31)$$

In a manner identical to that used to determine $P(AT_A)$ we have

$$P(BT_B) = \int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_3}^{x_3+z_B} dx \int_0^{z_B-(x-x_3)} dz \int_{z_B-z}^{b_{max}} db \quad P(i)$$

$$+ \int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_3}^{x_3+(B_m-z_B)} dx \int_{z_B+(x-x_3)}^{B_m} dz \int_{z-z_B}^{b_{max}} db \quad P(i)$$

$$+ \int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_3}^{x_3+z_B} dx \int_{z_B-(x-x_3)}^{z_B+(x-x_3)} dz \int_{2(x-x_3)}^{b_{max}} db \quad P(i)$$

$$+\int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_3+z_B}^{x_3+B_m-z_B} dx \int_0^{z_B+(x-x_2)} dz \int_{2(x-x_3)}^{b_{max}} db \quad P(i)$$

$$+\int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_3+B_m-z_B}^{L_B'} dx \int_0^{B_m} dz \int_{2(x-x_3)}^{b_{max}} db \quad P(i)$$

(32)

Now by substituting Equations (30), (31), and (32) into Equation (29), we can obtain the probability of loss of shaft B. The extension to P(SC), the probability of loss of shaft C, and to P(SD), the probability of loss of shaft D, follows merely by substituting z_C, x_4, L_C' and z_D, x_5, L_D' respectively into the above integrals.

4. Case of Loss of Two or More Shafts

The loss of two or more shafts from an underbottom explosion is perhaps the criteria most important in the design of an engineering plant with regard to the placement of engines and shafts. The knowledge of the pro-

bability for such a loss would be highly desirable and theoretically could lead to an optimum arrangement of shafting. (A simple case of this was shown in Figure XVIII). For the present we shall look at what the probability of loss of two shafts would be.

In order to simplify notation, we shall define the symbols to be used as follows:

A,B,C, and D: Each separately refers to the loss of the respective shaft whether the event occurs from the loss of the respective space or otherwise.

AB: Loss of shafts A and B together. In general ij represents the loss of shafts i and j together.

ABC: Loss of shafts A, B and C together. Similarly for any other combination of three symbols.

With these notations it should be apparent that the loss of two shafts results from the loss of AB or AC or AD or BC or BD or CD. What we want therefore is the probability of at least one of these combinations occurring. From probability theory with regards to the union of events,[7] it can be shown that

P(AB or AC or AD or BC or BD or CD) =

$$P(AB) + P(AC) + P(AD) + P(BC) + P(BD) + P(CD)$$
$$-2P(ABC) + 2P(ABD) - 2P(ACD) - 2P(BCD) + 3P(ABCD) \quad (33)$$

The actual determination of any one of these probabilities may be quite difficult if the desired expression is to be general or usable for any possible arrangement of shafting. To show this, we shall consider the case of determining P(AB).

In order to generalize the problem, one must first realize that shafts A and B could have two possible orientations with respect to each other, as shown in Figure XXI.

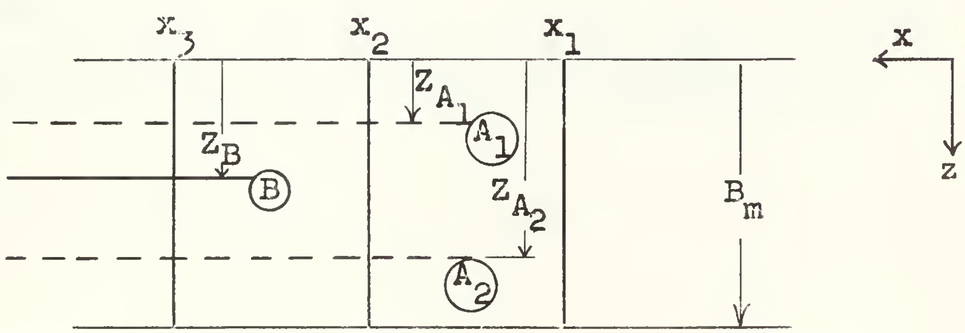


FIGURE XXI

Possible Arrangements of Shafts A and B

For the arrangement using A_1, $z_A < z_B$; whereas for the arrangement with A_2, $z_A > z_B$. If we consider either case, it is possible to lose both shafts A and B if any one of the following three events occurs:

1. A hit is the region of the separating bulkhead which results in the rupture of the bottom structure in both machinery spaces.

2. A hit solely under space B but under the region of B through which shaft A passes. This hit could damage shaft A without the loss of spáce A.

3. A hit aft of space B but capable of causing misalignment of both shafts in this region. This means misalignment so as to necessitate stopping the shaft. This event would normally occur only if shafts A and B are close enough to be misaligned from one hit.

Referring to Figure XXI, we shall take the case of shaft A and develop an integral expression for P(AB).

$$
\begin{aligned}
P(AB) = & \int_{0}^{w_{max}} dw \int_{0}^{y_{max}} dy \int_{0}^{x_2} dx \int_{0}^{B_m} dz \int_{2(x_2-x)}^{B_{max}} db \quad P(i) \\
& + \int_{0}^{w_{max}} dw \int_{0}^{y_{max}} dy \int_{x_2}^{x_2+z_A} dx \int_{0}^{z_A-(x-x_2)} dz \int_{2(x-x_2)}^{b_{max}} db \quad P(i) \\
& + \int_{0}^{w_{max}} dw \int_{0}^{y_{max}} dy \int_{x_2}^{x_2+(B_m-z_A)} dx \int_{z_A+(x-x_2)}^{B_m} dz \int_{2(x-x_2)}^{b_{max}} db \quad P(i) \\
& + \int_{0}^{w_{max}} dw \int_{0}^{y_{max}} dy \int_{x_2}^{x_2+z_A} dx \int_{z_4-(x-x_2)}^{z_A} dz \int_{2(z_A-z)}^{b_{max}} db \quad (P(i) \\
& + \int_{0}^{w_{max}} dw \int_{0}^{y_{max}} dy \int_{x_2+z_A}^{x_3} dx \int_{0}^{z_A} dz \int_{2(z_A-z)}^{b_{max}} db \quad P(i)
\end{aligned}
$$

$$+\int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_2}^{x_3} dx \int_{z_A}^{z_A+(x-x_2)} dz \int_{2(z-z_A)}^{b_{max}} db \quad P(i)$$

$$+\int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_3}^{x_2+B_m-z_A} dx \int_{\frac{z_A+z_B}{2}}^{z_A+(x-x_2)} dz \int_{2(z-z_A)}^{b_{max}} db \quad P(i)$$

$$+\int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_3}^{L} dx \int_0^{\frac{z_A+z_B}{2}} dz \int_{2(z_B-z)}^{b_{max}} db \quad P(i)$$

$$+\int_0^{w_{max}} dw \int_0^{y_{max}} dy \int_{x_2+B_m-z_A}^{L} dx \int_{\frac{z_A+z_B}{2}}^{B_m} dz \int_{2(z-z_A)}^{b_{max}} db \quad P(i) \tag{34}$$

The next to last integral is not exactly accurate in that in the region of z from 0 to z_A and x close to x_3,

the damage length could be somewhat less than $2(z_B - z)$. However an exact calculation would result in about six more integrals. The integral shown is sufficiently accurate so that the error would be extremely small.

With reference to Equation (33), this final expression for P(AB) is merely the first term. The remaining terms will be similar but may be even more complicated. Further difficulty is encountered when one considers the fact that there are twenty-four different arrangements for the four shafts, two of which are shown in Figure XVIII. Thus the problem rapidly becomes extremely complex when one attempts to solve for any probabilities involving more than one shaft. The determination of other desired probabilities follows by application of the preceding method and by proper use of probability theorems with regards to the algebra of events.

An illustrative example for the underbottom noncontact explosion case is shown in Appendix B.

IV. DISCUSSION AND RECOMMENDATIONS

This thesis has been an attempt to formulate a method of determining the probability of damage to a ship, specifically with regards to the loss of main machinery rooms and shafting of aircraft carriers. The method developed has been rather specific in that we require the knowledge of certain density functions, namely those for the distribution of hits in the three dimensions, for the distribution of damage lengths, and for the distribution of charge weights expected to be used against the ship. However the method is general in that it is applicable to any type of underwater protection system, whether it be a destroyer hull or the sophisticated system of an aircraft carrier.

Whether the method developed is actually suitable for use is entirely dependent upon the desired accuracy of the problem and upon the form of the density functions. The density functions postulated in the results do not lend themselves to easy evaluation but could be approximated by linear curves or polynomials such as the beta function.

The method is significant in that once the general expression for the loss of any particular number of spaces or shafts is determined, it might be possible to vary the parameters of the expression in an attempt to minimize the desired probability. For instance, in the case of the underbottom explosion a method of determining the probability of loss of two or more shafts was discussed. This probability would turn out to be a function of the position of the shafts from a reference point. By the application of calculus of variations utilizing Lagrange multipliers, it would be possible to determine what set of these parameters should be used to minimize the particular probability. It was pointed out that the determination of the general expression would be most difficult and require lengthy calculations. Nevertheless, from a mathematical viewpoint this is entirely possible.

When, in the determination of the probability of damage to a ship, one is required to resort to simple density functions such as impulses (as is done in Example 1, Appendix A), it becomes possible to solve for these probabilities by graphical means, which might be somewhat more easy an approach. An illustration of the method of graphical solution is shown in Example 3, Appendix A. However when the degree of difficulty is in-

creased only slightly to include uniform distributions, then the graphical approach becomes impossible and integration must necessarily be performed using the methods shown in the results.

In order to use these results, reasonable and realistic probabilities need to be determined. Therefore investigations would have to be made to determine the appropriate density functions, particularly with regards to the distribution of damage lengths as a function of charge weight and standoff distance. It is realized that an accurate determination of this distribution would require extensive and costly testing and might not be worth the effort and expense. However there does exist data which indicates what damage results to the longitudinal bulkheads of a side protection system from a charge exploded in contact with the side. This data has been accumulated from the series of caisson tests noted in references 21 and 22. In almost every case though, the system was tested only to the point where the holding bulkhead was deflected but not ruptured. Therefore the desired information, that is, the size of ruptures to the holding bulkhead, is lacking. A means of extrapolating the damage lengths occurring in the backing bulkheads to that which would occur in the holding bulkhead if

larger charge weights were used would be extremely useful and would provide the necessary data for the determination of the damage length density function.

Once this and the other density functions are determined, a program utilizing a digital computer possibly could be set up to evaluate the various probabilities should these functions be of a form not readily integrable. The adaptability of the integral equations formulated in the results to computer programming is discussed in Appendix D.

The authors believe that further work in this field would be desirable for the purpose of determining the optimum arrangement of main and auxiliary machinery spaces and shafting. Therefore it is recommended that the following steps be taken:

1. Investigation into the method of extrapolating damage lengths in the holding bulkheads from existing data on the damage to backing bulkheads.

2. Assessment of enemy capabilities to determine the types of underwater weapons expected to be used against an aircraft carrier and the most probable distribution of these weapons with regards to charge weight.

3. Investigation into the target hit capability of the various types of weapons.

4. Application of method of dynamic programming or calculus of variations to the results in order to optimize a machinery plant arrangement and minimize the probabilities of loss due to damage.

V. APPENDIX

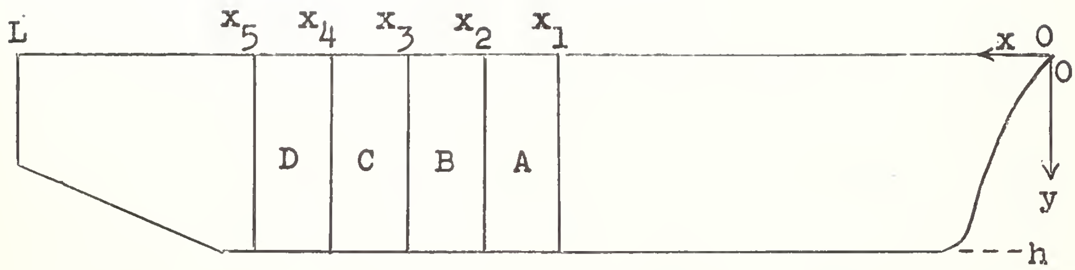
APPENDIX A

ILLUSTRATIVE EXAMPLES-SIDE CONTACT HIT

It has been the authors' intention to apply the results obtained to a typical machinery arrangement. Unfortunately quantitative data that would be necessary to accurately determine the required probability density function is either difficult to obtain or simply not available. It is therefore necessary to postulate what density functions will be used realizing that they may be broad approximations, but it is felt that this will be useful for providing an illustration of the limited use of this method. We will begin by assuming rather simple density functions and proceed to more complicated and more realistic density functions, showing what difficulties arise in attempting to achieve greater accuracy.

Example 1. Probability of Loss of Machinery Spaces - No Auxiliary Spaces

Consider the arrangement shown below in Figure XXII.



<u>FIGURE XXII</u>

Arrangement of Machinery Spaces

Assume the following density functions.

a) Uniform distribution for x and y

$$p(x) = \begin{cases} \frac{1}{L} & 0 \leqslant x \leqslant L \\ 0 & \text{elsewhere} \end{cases}$$

$$p(y) = \begin{cases} \frac{1}{h} & 0 \leqslant y \leqslant h \\ 0 & \text{elsewhere} \end{cases}$$

b) A fixed charge weight, w_o.

$$p(w) = U_o(w - w_o) = \begin{cases} 1 & w = w_o \\ 0 & \text{elsewhere} \end{cases}$$

c) A fixed damage length, b_o, for the fixed charge weight w_o.

$$p(b/w) = U_o(b-b_o) = \begin{cases} 1 & b = b_o \\ 0 & \text{elsewhere} \end{cases}$$

We will also assume that the damage length b_o is less than the length of any machinery space.

$$b_o < x_{i+1} - x_i \qquad i = 1,2,3,4$$

The probability of loss of just one space is carried out using Equations (11) through (14).

$$P(AB'C'D) = \int_0^{w_{max}} dw \int_0^h dy \left[\int_0^{x_1} dx \int_{2(x_1-x)}^{2(x_2-x)} db \quad U_o(b-b_o)\frac{1}{L}\cdot\frac{1}{h}\,U_o(w-w_o) \right.$$

$$\left. + \int_{x_1}^{x_2} dx \int_0^{2(x_2-x)} db \quad U_o(b-b_o)\frac{1}{L}\cdot\frac{1}{h}\,U_o(w-w_o) \right]$$

$$= \int_{x_1-b_o/2}^{x_1} dx \cdot \frac{1}{L} + \int_{x_1}^{x_2-b_o/2} dx \cdot \frac{1}{L}$$

$$P(AB'C'D') = \frac{1}{L}(x_2 - x_1)$$

Similarly

$$P(A'BC'D') = \frac{1}{L}(x_3 - x_2 - b_o)$$

$$P(A'B'CD') = \frac{1}{L}(x_4 - x_3 - b_o)$$

$$P(A'B'C'D) = \frac{1}{L}(x_5 - x_4)$$

Therefore

$$P(1) = \frac{1}{L}(x_5 - x_1 - 2b_o)$$

The probability of loss of no machinery space is the probability that the hit occurs outside the machinery spaces by an amount $b_o/2$ either side.

$$P(0) = \frac{1}{L}\left[L - (x_5 - x_1 + b_o)\right]$$

$$P(2 \text{ or more}) = 1 - P(1) - P(0)$$

$$= 1 - \frac{1}{L}(L - 3\,b_o)$$

$$P(2 \text{ or more}) = \frac{3b_o}{L}$$

Example 2. <u>Probability of Loss of Main Machinery Spaces - One Auxiliary Space</u>

Consider now the arrangement shown below in Figure XXIII, in which an auxiliary space is located between spaces B and C.

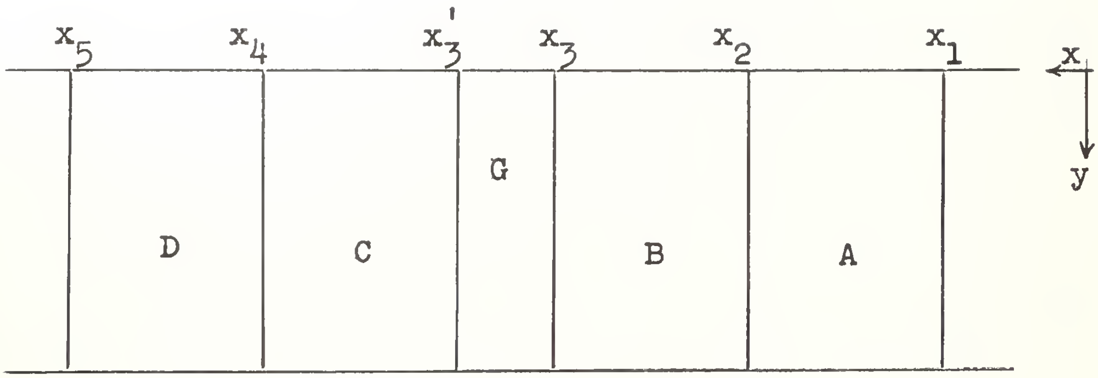


FIGURE XXIII

Arrangement of Machinery Spaces Including One Auxiliary Space

Calculations of the probability of loss of 2 or more spaces will be performed using the same density functions of Example 1.

$P(AB'C'D) = \frac{1}{L}(x_2 - x_1)$ (unchanged from ex. 1)

$$P(A'BC'D') = \begin{cases} \frac{1}{L}(x_3' - x_2 - b_o) & 0 < (x_3' - x_3) < b_o \\ \frac{1}{L}(x_3 - x_2) & b < (x_3' - x_3) \end{cases}$$

$$P(A'B'CD') = \begin{cases} \frac{1}{L}(x_4 - x_3 - b_o) & 0 < (x_3' - x_3) < b_o \\ \frac{1}{L}(x_4 - x_3') & b_o < (x_3' - x_3) \end{cases}$$

$P(A'B'C'D) = \frac{1}{L}(x_5 - x_4)$

$$P(1) = \begin{cases} \frac{1}{L}(x_5 + x_3' - x_3 - x_1 - 2b_o) & 0 < (x_3' - x_3) < b_o \\ \frac{1}{L}\left[x_5 - x_1 - (x_3' - x_3)\right] & b_o < (x_3' - x_3) \end{cases}$$

$$P(0) = \begin{cases} \frac{1}{L}\left[L - (x_5 - x_1 + b_o)\right] & 0 < (x_3' - x_3) < b_o \\ \frac{1}{L}\left[L - (x_5 - x_1 + d_o) + (x_3' - x_3) - b_o\right] & b_o < (x_3' - x_3) \end{cases}$$

$$P(2) = 1 - P(1) - P(0)$$

$$P(2) = \begin{cases} \frac{3b_o - (x_3' - x_3)}{L} & 0 < (x_3' - x_3) < b_o \\ \frac{2\,b_o}{L} & b_o < (x_3' - x_3) \end{cases}$$

A plot of P(2) as a function of the length of the auxiliary space, $(x_3' - x_3)$, is shown in Figure XXIV. It also includes P(2) for Example 1 to indicate the relation between the two examples.

Figure XXIV indicates the manner in which the probability of loss of two or more spaces changes as the length of the auxiliary space increases. The results are as expected. If the separation between two main machinery

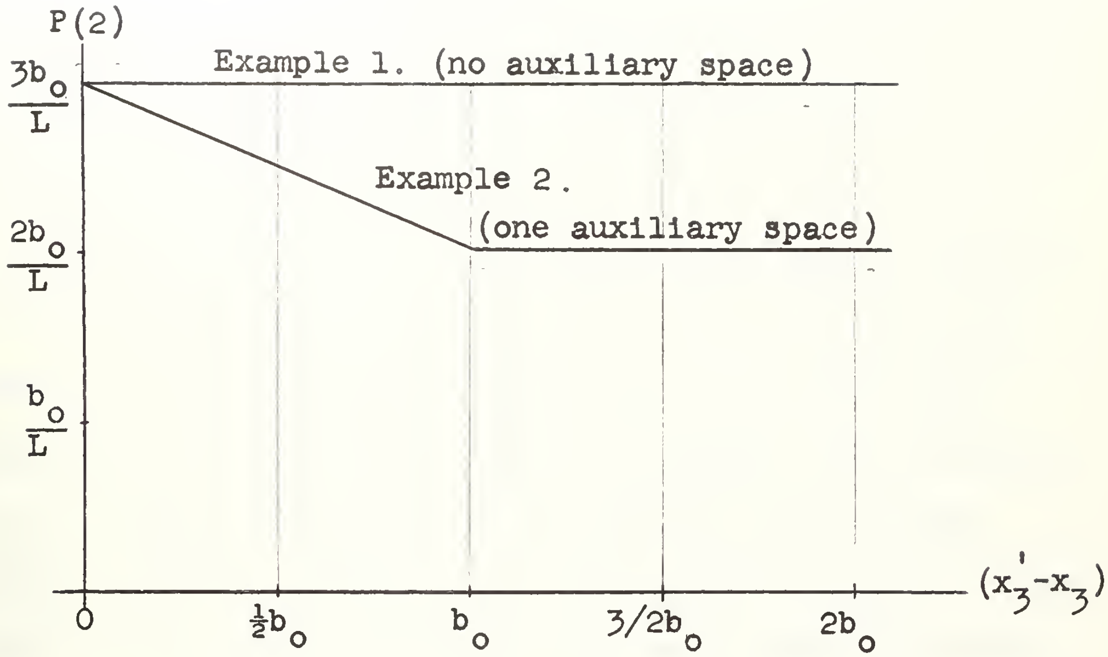


FIGURE XXIV

Probability of Loss of Two or More Machinery Spaces For Case of None and One Auxiliary Space

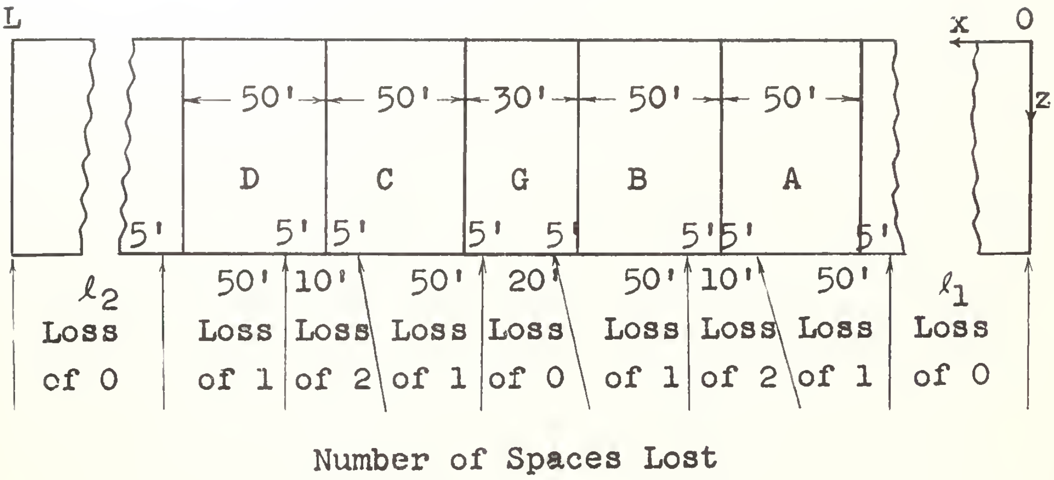
spaces increases there is less chance of losing both spaces. Because the density function for P(b/w) was an impulse indicating only one possible damage length, the probability decreases linearly until the length corresponds to the damage length. It should be noted that because of the increase in length of the machinery box, there is a resultant increase in the probability of losing one space.

Example 3. <u>Probability of Loss of Main Machinery Spaces - Graphical Method</u>

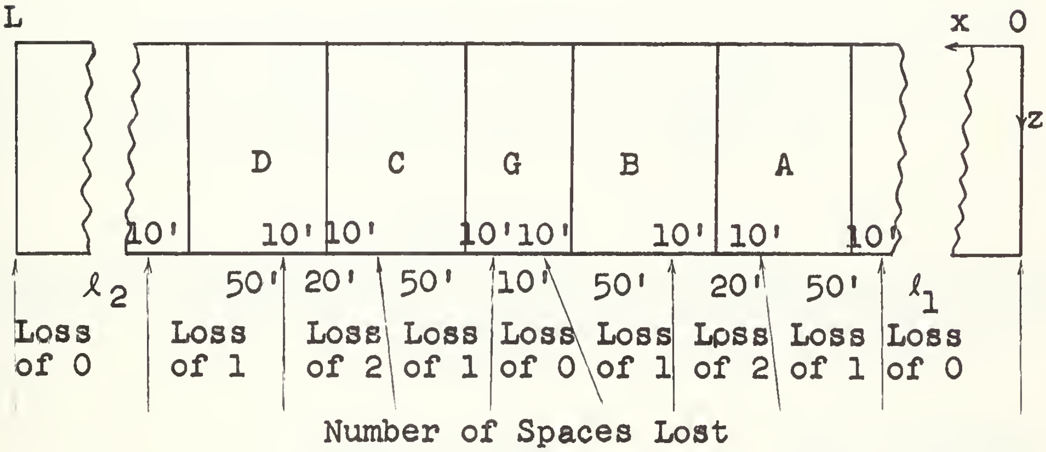
In this example we shall again consider the arrangement shown in Figure XXIII,,where there is an auxiliary space located between the middle two machinery spaces. Graphical methods will be used to determine the probable loss of just the main machinery spaces given a fixed damage length. Hence if just the auxiliary space is lost, then it will be considered as zero main machinery spaces lost. Two calculations will be made;one for a fixed damage length b of 10 feet and one for a fixed damage length of 20 feet.

For a side contact hit, the determination of probable losses by graphical methods simply requires the movement of the fixed damage length along the side of the ship and noting for each location the number of machinery spaces that would be lost. Then to determine the probability of loss of i spaces (i = 0,1,2,3, or 4), it merely requires that the various lengths of i spaces lost be summed up and divided by the total length.

The arrangement used and the graphical solution for the two fixed damage lengths are shown in Figure XXV. Note that since the damage lengths are less than the lengths of the spaces, then the maximum number of spaces that can be lost is two. To obtain numerical results,



A. Fixed Damage Length = 10 ft.



B. Fixed Damage Length = 20 ft.

FIGURE XXV

Graphical Determination of Loss of Spaces

A. Fixed damage length = 10 ft.

B. Fixed damage length = 20 ft.

consider the length of the ship, L, to be 1000 feet.

For case A with $i = 1$, the calculation to determine P(1), the probability of the loss of 1 main machinery space, is as follows:

$$t_1 = \sum (\text{loss of 1 space}) = 50 + 50 + 50 + 50$$
$$= 200 \text{ ft.}$$

$$P(1) = t_1/L = \frac{200}{1000} = 0.20$$

The calculations to determine the other probabilities for cases A and B are carried out in a similar manner. These results are summarized below in Table A-1.

TABLE A-1

Probabilities of Loss of Spaces as Determined by Graphical Methods

	A. b = 10 ft.		B. b = 20 ft.	
	t_1 (ft.)	P(i)	t_1 (ft.)	P(i)
Loss of 0 spaces	780	0.78	760	0.76
Loss of 1 space	200	0.20	200	0.20
Loss of 2 spaces	20	0.02	40	0.04

Example 4. Application of Beta Functions to Represent P(b/w)

As brought out in the body of the paper it appears that a realistic density function for P(b/w) might be something similar to a family of beta functions. It was also mentioned that the mean damage length might appear as some function of w in the form $\bar{b} = k_o(w-w_c)^{\gamma_o}$. The beta function has the form

$$f(x) = \frac{1}{C}(x - A)^{\alpha} \cdot (B - x)^{\lambda} \qquad A \leqslant x \leqslant B$$

$$C = (B-A)^{\alpha + \lambda + 1} \qquad \frac{\Gamma(\alpha +1)\ \Gamma(\lambda +1)}{\Gamma(\alpha + \lambda + 2)}$$

The mean of this function in the range A,B is dependent upon the constant parameters α and λ. For $\alpha = \lambda$ the mean is at $\frac{A+B}{2}$ and the function is symmetric about the mean. In order to arrive at some expression for P(b/w) we will assume the following parameter values:

$$k_o = 2$$

$$\gamma_o = 1$$

$$\alpha = \lambda = 2$$

$$B = A + 4$$

$$A = 2(w - w_c) - 2$$

$$B = 2(w - w_c) + 2$$

$$C = \frac{(16)(32)}{15}$$

With these values, the density function for the distribution of damage lengths is

$$P(b/w) = \frac{15}{(16)(32)} \left[b - (2(w-w_c)-2)\right]^2 \left[2(w-w_c)+2-b\right]^2 .$$

A plot of $P(b/w)$ for various values of w is shown in Figure XXVI. Here the assumption is made that $w_c = 15$ representing a system capable of defeating a 1500 lb. charge. The symbol w will designate a charge weight in hundreds of pounds.

We will assume that the density functions for $P(x)$ and $P(y)$ are uniform in the ranges 0 to L and 0 to h respectively. The discussion has brought out the fact that the normal density function cannot be integrated explicitly between finite limits, a limitation restricting the actual use of the normal considerably.

$$\text{Let } P(w) = \begin{cases} \frac{1}{4} & 16 \leqslant w \leqslant 20 \\ 0 & \text{elsewhere} \end{cases}$$

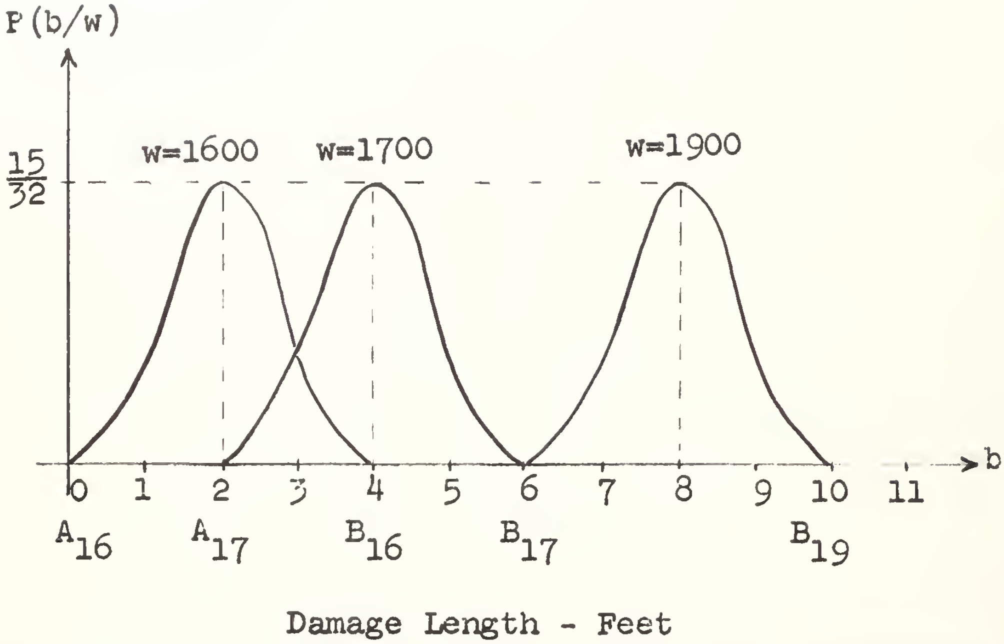


FIGURE XXVI

Plot of Family of Beta Functions

With these assumptions we will proceed to evaluate the probability of the loss of two spaces for the arrangement given in Figure XXII. The expressions to be used for these calculations are given in Equations (17) through (19).

$$P(ABC'D') = \int_{16}^{20} dw \int_{o}^{h} dy \int_{o}^{x_2} dx \int_{2(x_2-x)}^{2(x_3-x)} db \quad \frac{1}{C}(b-A)^2(B-b)^2 \frac{1}{L\cdot h\cdot 4}$$

$$+ \int_{16}^{20} dw \int_{o}^{h} dy \int_{x_2}^{\frac{x_3+x_2}{2}} dx \int_{2(x-x_2)}^{2(x_3-x)} db \quad \frac{1}{C}(b-A)^2(B-b)^2 \frac{1}{L\cdot h\cdot 4}$$

The integration with respect to b can have different values dependent upon where the limits of b fall. For instance in the first integral, the lower limit of d may be less than A, between A and B, or greater than B. Caution must be exercised to insure that the correct limits are applied to the various regions of interest.

$$P(ABC'D') = \int_{16}^{20} dw \int_{x_2-X/2}^{x_2} dx \int_{A}^{B} db \quad P(b/w) \frac{1}{4L}$$

$$+ \int_{16}^{20} dw \int_{x_2-B/2}^{x_2-A/2} dx \int_{2(x_2-x)}^{B} db \quad \frac{15}{(16)(32)} \frac{1}{4L} (b-A)^2(B-b)^2$$

$$+\int_{16}^{20} dw \int_{x_2}^{x_2+A/2} dx \int_{A}^{B} db \; P(b/w) \frac{1}{4L}$$

$$+\int_{16}^{20} dw \int_{x_2+A/2}^{x_2+B/2} dx \int_{2(x-x_2)}^{B} db \frac{15}{(16)(32)} \cdot \frac{1}{4L} (b-A)^2(B-b)^2 \tag{39}$$

The first and third integrals simplify considerably because of the fact that the integral with respect to b is 1. The remaining two integrals on b are not simple at all although they do lend themselves to formal integration.

$$\int_{2(x_2-x)}^{B} db\,(b-A)^2(B-b)^2$$

$$= \int_{2(x_2-x)}^{B} \left\{ b^4 - 2(A+B)b^3 + \left[(A+B)^2 + 2AB\right] b^2 - 2AB(A+B)b + A^2 + B^2 \right\} db$$

$$= \left[\frac{b^5}{5} - \frac{2(A+B)}{4} b^4 + \left[\frac{(A+B)^2 + 2AB}{3}\right] b^3 - \frac{2AB(A+B)}{2} b^2 + (A^2 + B^2)b \right]_{2(x_2-x)}^{B}$$

The limits above apply to the second term whereas the limits B and $2(x-x_2)$ apply to the fourth term of Equation (39). The result of this evaluation will be a function that is fifth order in x and w. This resultant must then be integrated with respect to x between limits which are functions of w now. This result must then be integrated with respect to w and finally evaluated. The result is a number which is only a portion of the final answer to the problem. It is quite apparent that the use of a beta function of relatively low order ($\alpha = \lambda = 2$) results in an expression quite difficult or at least tedious. The problem would become quite out of hand if a normal density function were substituted for P(x) or a gamma function for P(y).

A complete evaluation of the integrals in Equation (39) would serve no useful purposes at present. It clearly points out the extreme difficulty one encounters in using some realistic density functions. Once one leaves the realm of discrete functions (unit impulses) and uniform density functions, the problem becomes enormous.

The three examples indicate what procedure must be followed in order to evaluate the relatively simple case of loss of one and two shafts from a side hit, and

also the problem that arises in attempting to make the problem more sophisticated but hopefully more realistic and accurate.

APPENDIX B

ILLUSTRATIVE EXAMPLE - UNDERBOTTOM NON-CONTACT EXPLOSION

As we did in Appendix A for the case of the side contact hit, we shall now postulate some probability density functions for use in the underbottom non-contact explosion and calculate a probability of damage. Again these will be rather simple density functions in order that the calculations may be carried out.

Example 1. Probability of Loss of Machinery Spaces - No Auxiliary Spaces

Consider the arrangement shown below in Figure XXVII.

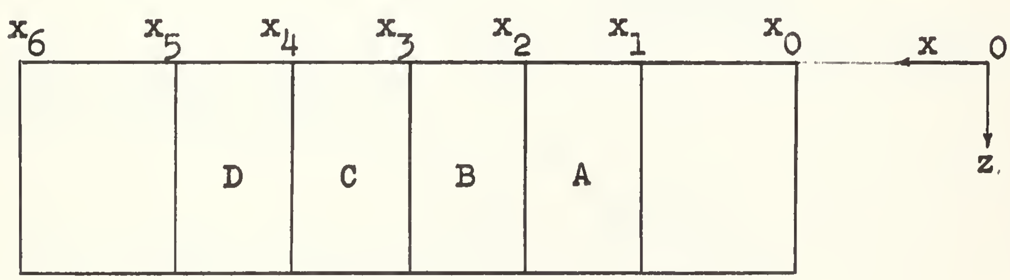


FIGURE XXVII

Arrangement of Machinery Spaces

Assume the following density functions:

a) Uniform distribution for x in the range x_o to x_6, where the underbottom protection system has constant beam.

$$P(x) = \begin{cases} \dfrac{1}{x_6 - x_o} & x_o < x < x_6 \\ 0 & \text{elsewhere} \end{cases}$$

b) Uniform distribution for z

$$P(z) = \begin{cases} \dfrac{1}{B_m} & 0 < z < B_m \\ 0 & \text{elsewhere} \end{cases}$$

c) A fixed depth, y_o

$$P(y) = U_o(y - y_o) = \begin{cases} 1 & y = y_o \\ 0 & \text{elsewhere} \end{cases}$$

d) A fixed charge weight, w_o

$$P(w) = U_o(w - w_o) = \begin{cases} 1 & w = w_o \\ 0 & \text{elsewhere} \end{cases}$$

e) Uniform distribution for damage length b in the range b_1 to b_2 for the fixed charge weight w_o and fixed depth y_o.

$$P(b/w_o,y_o) = \begin{cases} \dfrac{1}{b_2-b_1} & b_1 \leqslant b \leqslant b_2 \\ 0 & \text{elsewhere} \end{cases}$$

Let A = Event of loss of machinery space A

Then P(A) will be evaluated using Equation (23)

$$P(A) = \int_0^{w_{max}} dw \int_0^{B_m} dz \int_0^{y_{max}} dy \int_{x_1}^{x_2} dx \int_0^{b_{max}} db \left[\left(\frac{1}{b_2-b_1}\right)\left(\frac{1}{x_6-x_o}\right) \cdot U_o(y-y_o) \cdot \frac{1}{B_m} U_o(w-w_o)\right]$$

$$+ \int_0^{w_{max}} dw \int_0^{B_m} dz \int_0^{y_{max}} dy \int_0^{x_1} dx \int_{2(x_1-x)}^{b_{max}} db \left[\left(\frac{1}{b_2-b_1}\right)\left(\frac{1}{x_6-x_o}\right) \cdot U_o(y-y_o) \cdot \frac{1}{B_m} U_o(w-w_o)\right]$$

$$+ \int_0^{w_{max}} dw \int_0^{B_m} dz \int_0^{y_{max}} dy \int_{x_2}^{L} dx \int_{2(x-x_2)}^{b_{max}} db \left[\left(\frac{1}{b_2-b_1}\right)\left(\frac{1}{x_6-x_o}\right) \cdot U_o(y-y_o) \cdot \frac{1}{B_m} U_o(w-w_o)\right]$$

$$= \int_{x_1}^{x_2} dx\left(\frac{1}{x_6-x_o}\right) + \int_{x_1-\frac{b_1}{2}}^{x_1} dx\left(\frac{1}{x_6-x_o}\right) + \int_{\frac{x_1-b_2}{2}}^{x_1-\frac{b_1}{2}} dx\left(\frac{1}{x_6-x_o}\right)\left(\frac{b_2-2(x_1-x)}{b_2-b_1}\right)$$

$$+ \int_{x_2}^{x_2+\frac{b_1}{2}} dx\left(\frac{1}{x_6-x_o}\right) + \int_{x_2+\frac{b_1}{2}}^{x_2+\frac{b_2}{2}} dx\left(\frac{1}{x_6-x_o}\right)\left(\frac{b_2-2(x-x_2)}{b_2-b_1}\right)$$

$$= \frac{1}{x_6-x_o}\left\{(x_2+\frac{b_1}{2})-(x_1-\frac{b_1}{2}) + \frac{b_2}{b_2-b_1}\left[(x_1-\frac{b_1}{2})-(x_1-\frac{b_2}{2})+(x_2+\frac{b_2}{2})-(x_2+\frac{b_1}{2})\right]\right.$$

$$- \frac{2x_1}{b_2-b_1}\left[(x_1-\frac{b_1}{2})-(x_1-\frac{b_2}{2})\right] + \frac{1}{b_2-b_1}\left[(x_1-\frac{b_1}{2})^2-(x_1-\frac{b_2}{2})^2\right]$$

$$\left.- \frac{1}{b_2-b_1}\left[(x_2+\frac{b_2}{2})^2-(x_2+\frac{b_1}{2})^2\right] + \frac{2x_2}{b_2-b_1}\left[(x_2+\frac{b_2}{2})-(x_2+\frac{b_1}{2})\right]\right\}$$

Simplification of this leads to the final result.

$$P(A) = \frac{1}{x_6-x_o}\left[(x_2-x_1) + \frac{b_1+b_2}{2}\right]$$

In a similar way, the probability of loss of spaces B, C, and D can be found. Hence this example shows the validation of using this method of solution for the underbottom non-contact explosion case. Naturally if

more realistic probability density functions were used, such as that used for $P(b/w)$ in Example 3 of Appendix A, a more accurate answer would have been attained. However the calculations would have been much more difficult. Since the aim of the illustrative example is not to obtain a high degree of accuracy but rather to show the validation of using this probabilistic approach, it is felt that this example and those in Appendix A are sufficient proof.

APPENDIX C

DETERMINATION OF PROBABILITY DENSITY FUNCTION P(X)

In order to determine what a probable distribution of hits would be along the length of a ship, the approximate locations of actual torpedo hits from submarine attacks against large ships during World War II were obtained from references 8 to 11. These locations are listed in Table C-1 as a percentage of the length of the ship aft of the forward perpendicular. The number of hits in each two percent range of length is plotted in Figure XXVIII.

Fortunately for the U. S. Navy, but unfortunately for this paper, the number of hits was not enough to give sufficient data for an accurate plot. However the curve does appear to be similar to that of a normal density function, with the mean located approximately admiships. This is reasonable since it is logical to assume that the submarine attacks during World War II were conducted using optical aiming, with the point of aim being the midship section of the target. Since normal torpedoes, as opposed to homing weapons, were used, it can be assumed that they would normally travel along the line of fire. That hits occurred other than at amidships can be attributed to many

factors, such as inaccurate attack solutions, faulty torpedo mechanisms, or maneuvering of the target after the torpedo was fired and before the weapon hit the ship.

TABLE C-1

Ships Damaged or Lost Due to Submarine Torpedo Attack

Ship	Location of Hit Aft of FP (% of L)
USS SARATOGA (CV3)	50.0 61.9
USS YORKTOWN (CV5)	43.6 47.2
USS QUINCY (CA39)	30.5 52.0
USS VINCENNES (CA44)	27.0
USS WASP (CV7)	20.9 36.0
USS NORTHAMPTON (CA26)	73.3 79.0
USS ATLANTA (CL51)	42.3
USS JUNEAU (CL52)	42.3
USS NORTH CAROLINE (BB55)	25.2
USS CHESTER (CA27)	50.5
USS NEW ORLEANS (CA32)	28.5
USS PENNSACOLA (CA24)	71.8
USS LISCOME BAY (CVE56)	80.0
USS BLOCK ISLAND (CVE21)	5.3 89.0 73.7
USS RENO (CL96)	69.2
USS INDIANAPOLIS (CA35)	5.3 30.3

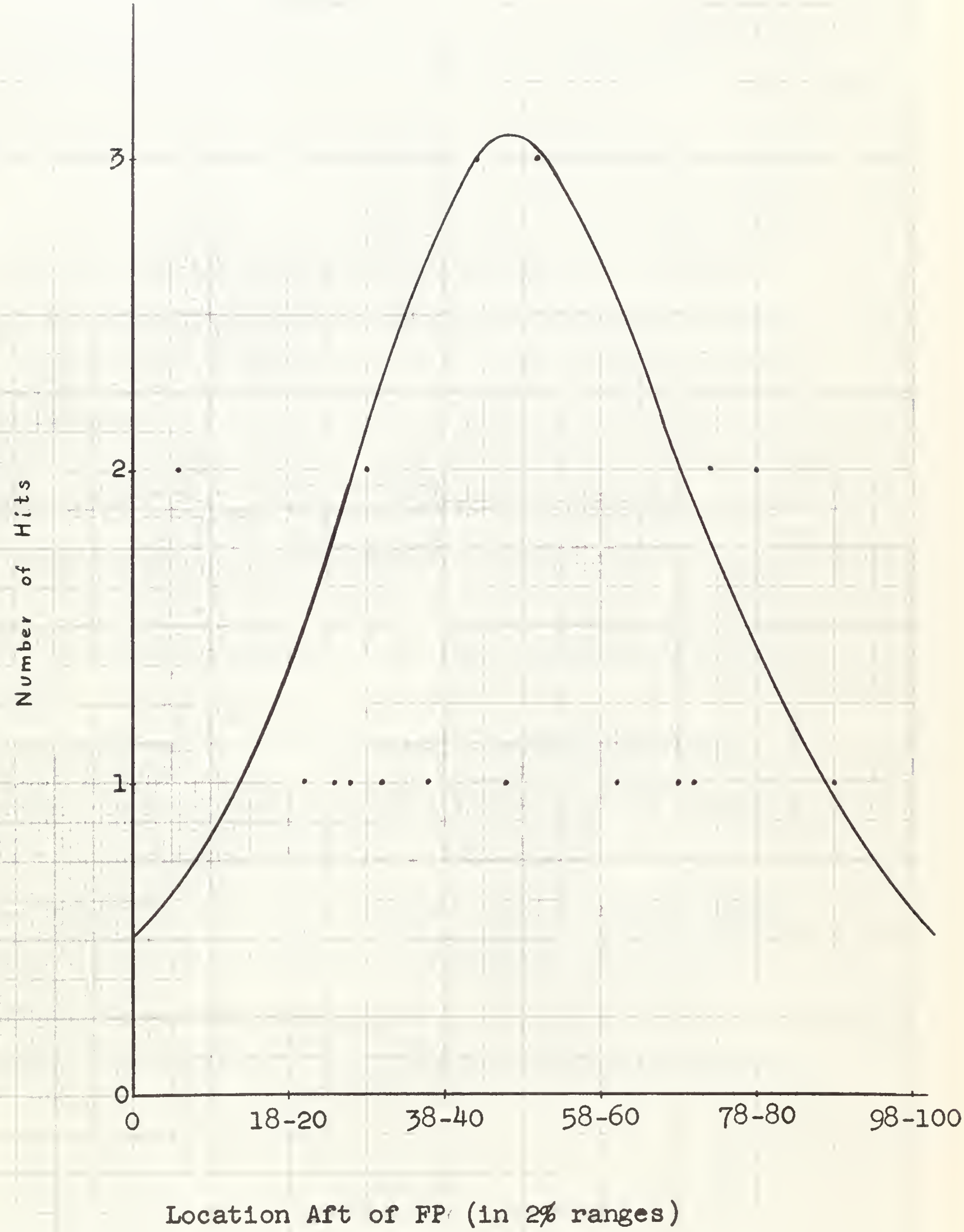


FIGURE XXVIII

DISTRIBUTION PATTERN OF TORPEDO HITS AGAINST LARGE SHIPS IN WORLD WAR II

APPENDIX D

ADAPTABILITY OF RESULTS TO COMPUTER PROGRAMMING

The integral equations arrived at in the results might not be integrable for integrands which are functions such as the normal or gamma. Each of these functions by itself can be evaluated only by numerical methods and then only between fixed numerical limits. The integration of a gamma function,

$$h(y) = \frac{y^{\alpha} e^{-y/\beta}}{\beta^{\alpha+1} \Gamma(\alpha+1)}$$

between functional limits such as f(x) and g(x) would be impossible.

Nevertheless, for the integral equations such as those found in the results of this thesis, it is possible to resort to numerical evaluation through the use of digital computers. How and why this can be accomplished can best be seen by considering an example of such an integration.

Assume one desired to evaluate the following equation,

$$A = \int_{a}^{b} dx \int_{f(x)}^{g(x)} dy \;\; p(y)h(x) \qquad (35)$$

where $f(x)$, $g(x)$, $p(y)$, and $h(x)$ are all known functions of their respective variables, and "a" and "b" are fixed numerical constants.

Expressed in words, Equation (35) states that the area under $p(y)$ between $f(x_i)$ and $g(x_i)$ is to be weighted by the product $h(x_i)\Delta x$ and summed up over all possible values of x_i. In the continuous case the x_i's are an infinitesimal distance, dx vice Δx, apart and the summation reduces to an integral. An approximation to the continuous case can be made by assuming each x_i to be separated by a finite amount Δx. In this case the product $h(x_i)\Delta x$ would appear as the shaded area in Figure XXIX.

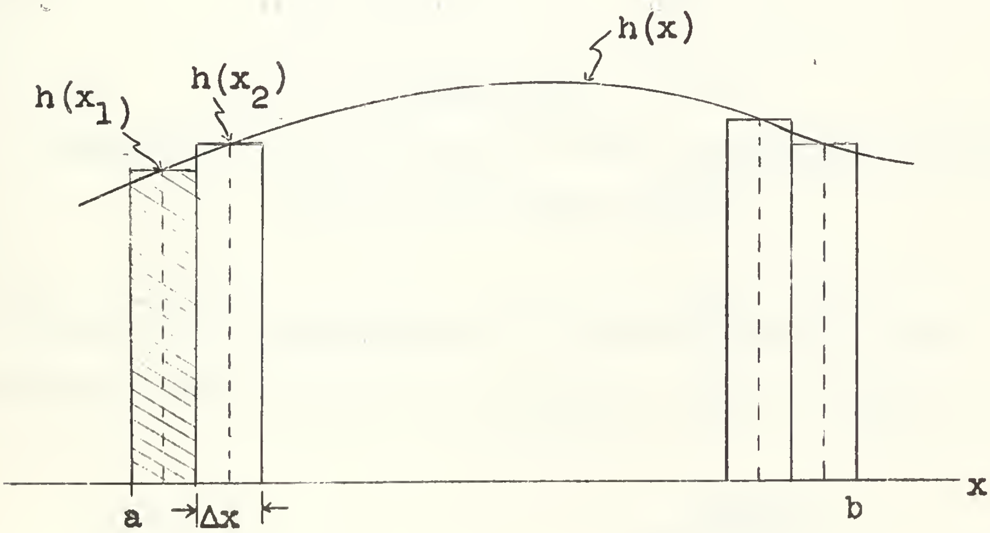


FIGURE XXIX

Discrete Approximation to h(x)dx

The value of the function $f(x_i)$ in the discrete interval Δx is taken as the value at the midpoint of Δx. The area under $p(y)$ appropriate for particular values of the functions $f(x)$ and $g(x)$ is shown in Figure XXX.

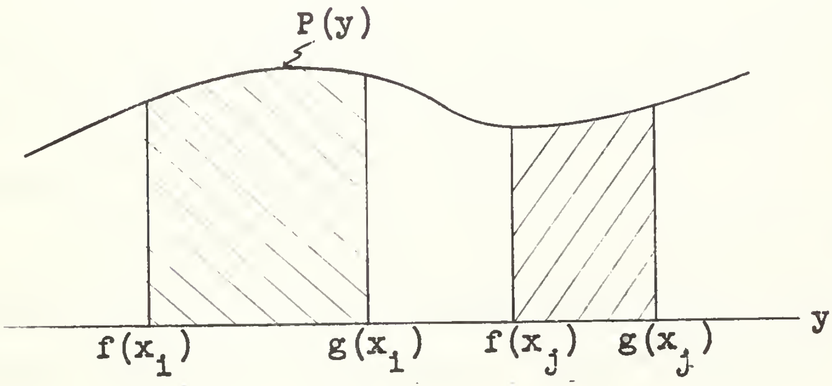


FIGURE XXX

Graphical Representation of the Integration of p(y) Between Functional Limits

Using this approximation to the continuous case, the value of the integral of Equation (35) is found to be

$$A = \sum_{j=0}^{\left(\frac{b-a}{\Delta x}\right)-1} h\left(a+\frac{(2j+1)}{2}\Delta x\right)\cdot\Delta x \int_{f\left(a+\frac{2j+1}{2}\Delta x\right)}^{g\left(a+\frac{2j+1}{2}\Delta x\right)} p(y)dy \qquad (36)$$

The limits of p(y) are now finite numerical constants and the integral of p(y) can be readily calculated by use of a digital computer. This value, the area shown in Figure XXX, is then multiplied by the appropriate weighting factor as indicated in Equation (36) and this product is stored. The cycle is then repeated for the new $x_1 = a + \frac{(2j+1)}{2}\Delta x$. The end result is a series of numbers which when summed yield an approximation for A, (Equation (36)). As the Δx's are made smaller, the approximation becomes more accurate.

If the expression to be evaluated involves three integrals such as shown in Equation (37), then the following procedure can be used. Select discrete

$$B = \int_C^d dw \int_a^b dx \int_{f(x)}^{g(x)} dy \;\; p(y/w)h(x)t(w) \tag{37}$$

intervals for w in a manner similar to that shown for x in the previous discussion.

Evaluate the double integral,

$$\int_a^b dx \int_{f(x)}^{g(x)} dy \;\; p(y/w_1)h(x)$$

as indicated by Equation (36) for a particular value of $w = w_i$. Weight this value by $t(w_i)\Delta w$, then proceed through the same cycle with the next value of w. The closed form expression for B in the discrete case is therefore given by Equation (38).

$$B = \sum_{i=0}^{\frac{d-c}{\Delta w}-1} t\left(c+\frac{(2i+1)}{2}\Delta w\right)\Delta w \left[\sum_{j=0}^{\frac{b-a}{\Delta x}-1} h\left(a+\frac{(2j+1)}{2}\Delta x\right)\Delta x. \int_{f(a+\frac{2j+1}{2}\Delta x)}^{g(a+\frac{2j+1}{2}\Delta x)} p(y/w_i)\,dy\right] \qquad (38)$$

where $\quad w_i = c + \frac{2i+1}{2}\Delta w \qquad 0 < i < \frac{d-c}{\Delta w} - 1$

By the approach discussed above, the evaluation of the probabilities brought out in the results can be adapted to computer programming.

APPENDIX E

COMBINED VERSUS SEPARATE SPACES

From the theory previously discussed, it can be shown that those aircraft carriers having combined machinery spaces will suffer less shafts lost due to a side or under-bottom explosion than will those having separate spaces. However when an internal fire or flooding is considered, the resulting damage and/or personnel casualties may lead to a completely different outcome of the value of one arrangement over the other.

When an internal casualty occurs in a space, the ability of the personnel to respond and their efficiency of action, as well as the ability of the Damage Control Parties, must be taken into consideration when trying to determine the extent of damage and injuries to be expected. Numerous factors must be taken into account when comparing the advantages and disadvantages of combined and separate arrangements. For example in the case of a fire which is not quickly extinguished, the combined space arrangement would probably cause more equipment to be put out of action, since in actuality more equipment is present in a combined space, and it could lead to greater

personnel casualties since more men would be in the combined space than in any single space of the separate arrangement.

In considering the efficiency of the personnel on watch, it can be thought of as being a random variable which is a function of time. At the beginning of the watch, the person would have to become acclimated to the conditions in the space and hence his efficiency would be low. As the watch progresses, his efficiency would increase to some maximum value and then decrease with time. Hence at the beginning or near the end of the watch period, a person's ability to react to various situations would be less. To incorporate this into a probabilistic approach, therefore, a suitable density function must be determined.

The state of training of the personnel could also be thought of as another random variable which varies with time. Again the ability to react would be less at the start and would increase as the training and experience increase. Although the state of readiness should reach a maximum and stay there, it would actually decrease since "familiarity breeds contempt." As some actions become common, they will be disregarded, and could lead to a casualty. This was shown in the case of the flooding

of MMR3 on board the USS FORRESTAL (CVA-59) on 23 December 1961.[13] According to the Ship' Engineering Instruction 9480.1, the Engineering Supervisor of the Watch shall obtain permission from the Engineering Duty Officer prior to pumping bilges in port and shall ensure that the cold iron watch is supervised while doing it. But it had become common practice not to do this on the FORRESTAL and hence it contributed to causing the eventual flooding of MMR3. Thus a suitable density function for state of training must be determined.

During 1961 there were several cases of fire and one of flooding aboard ships having combined machinery spaces. Neglecting human error which contributed to these cacualties occurring, it was the location of the oil lines that was the factor leading to the fires. In each case the fire was confined to one side of the MMR and it was the large amount of smoke produced which caused the fatalities on board the USS CONSTELLATION (CVS-64).[14] Hence the fact that the MMR's were combined spaces did not cause additional fire damage. Therefore having separate spaces would not have reduced this damage.

It would appear that a study of the aforementioned casualties, as given in references 12 to 14, would be of slight use in attempting to resort to a probabilistic approach in this area. Instead detailed studies would have

to be undertaken in order to determine the proper density functions for the many variables, such as those mentioned previously, if a probability format were to be developed to determine the effectiveness of combined and separate spaces.

The use of separate spaces would have reduced the damage in the case of flooding on board the USS FORRESTAL (CVS-59).[13] The separate spaces could be formed by using a design of separate fire rooms and engine rooms or by altering the arrangement in the combined spaces in order to install a longitudinal centerline bulkhead (see Figure XXXI). In either case the volume of a space is significantly reduced, so that flooding of a space from an internal casualty or from entering seas through a ruptured holding bulkhead would give a greatly reduced added displacement. In addition, the arrangement using a longitudinal centerline bulkhead generally would insure that some propulsion power would be available should all the spaces on one side of the ship be lost. However, with this arrangement a sizeable list would occur prior to counter-flooding. Examples of the amount of list resulting from the loss of two or four combined spaces are calculated below.

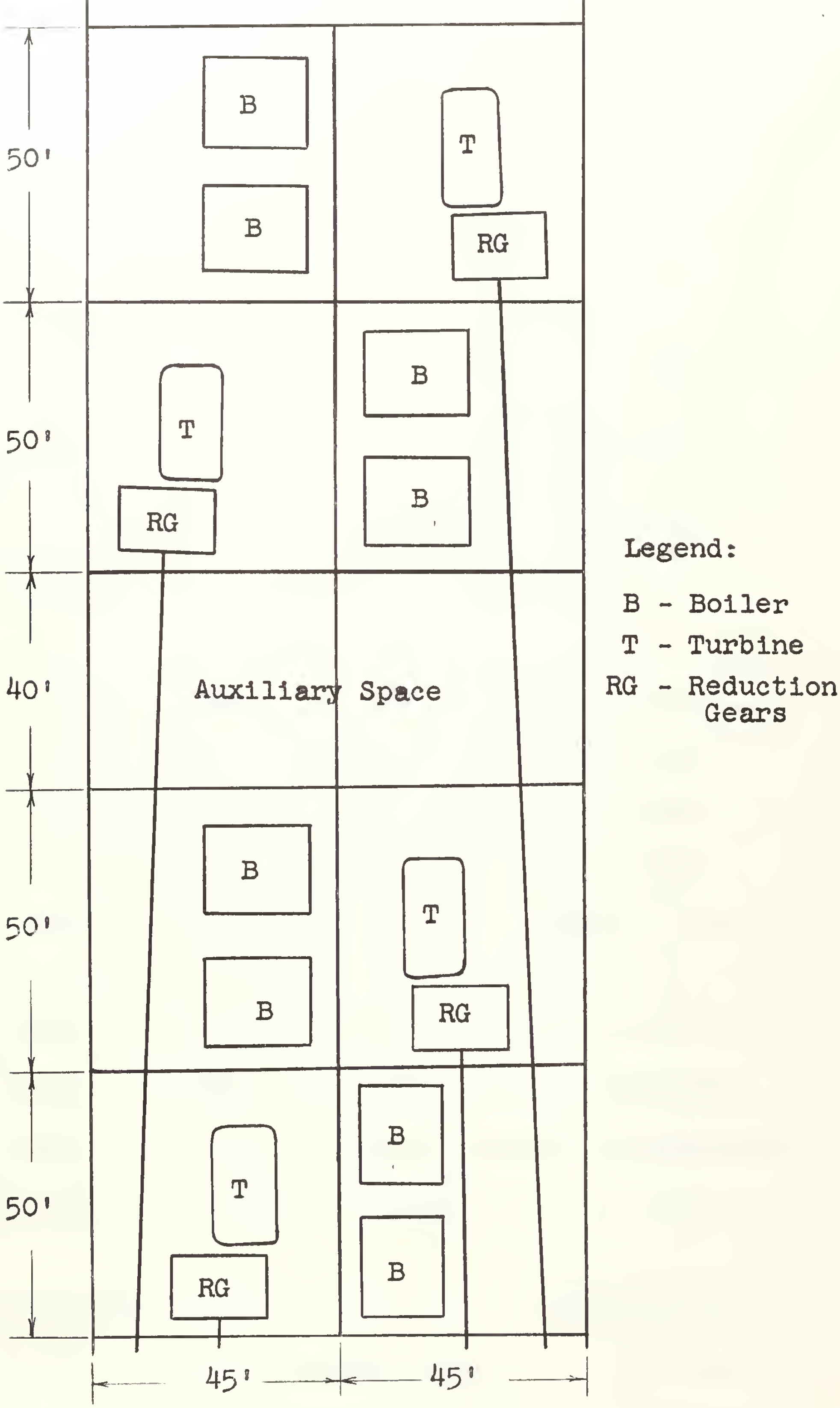


FIGURE XXXI

Arrangement of Machinery Spaces with a Longitudinal Centerline Bulkhead

Examples of List Occurring in a Machinery Arrangement Utilizing a Longitudinal Centerline Bulkhead.

As mentioned above, the use of a centerline bulkhead in the machinery spaces would be effective in reducing the extent of internal fire and flooding. It would also be instrumental in providing some propulsive power should the ship suffer hits along one side which would cause the loss of those portions of the spaces subjected to the attack.

The following calculations are performed in order to determine what list would occur should such large off-centerline spaces be flooded. These calculations are carried out assuming that the resultant trim will have negligible effect on the values of list. Actually since the machinery spaces are located aft of midship, the flooding of these spaces will result in a trim by the stern which would increase the inertia of the waterplane and tend to make the results obtained somewhat conservative.

The following conditions are assumed for the examples:

Displacement, full load	76,000 tons
Vertical Center of Gravity, KG	44 feet
Transverse Metacetric Height, GM	12 feet
Draft, full load	34 feet

The following results were obtained by a two-step iterative process utilizing information contained partially in classified documents (references 20 and 23); hence intermediate steps are not shown:

1. Flooding of Two (2) Main Machinery Spaces
   (see Figure XXXII)

a) First approximation:

Weight of added water .... 5,200 tons

Total displacement ....81,200 tons

Heeling moment due to added weight 158,200 ft.-tons

Heeling arm .... 1.95 ft.

Angle of list .... 8.5°

b) Second approximation:

Assumed list .... 10°

Weight of added water .... 6,100 tons

Total displacement .... 82,100 tons

Heeling moment due to added weight 198,500 ft.-tons

Heeling arm .... 2.42 ft.

Angle of list .... 10°

Since this calculated value of angle of list is equal to the assumed list of 10°, this is the approximate list of the ship with two machinery spaces flooded and before counter-flooding is undertaken.

2. Flooding of Four (4) Main Machinery Spaces
(see Figure XXXIII)

a) First approximation:

Weight of added water .... 10,400 tons

Total displacement .... 86,400 tons

Heeling moment due to added weight 316,500 ft.-tons

Heeling arm .... 3.67 ft.

Angle of list........................ 13.5°

b) Second approximation:

Assume list .... 16.0°

Weight of added water .... 12,700 tons

Total displacement .... 88,700 tons

Heeling moment due to added weight 408,100 ft.-tons

Heeling arm .... 4.60 ft.

Angle of list .... 15.5°

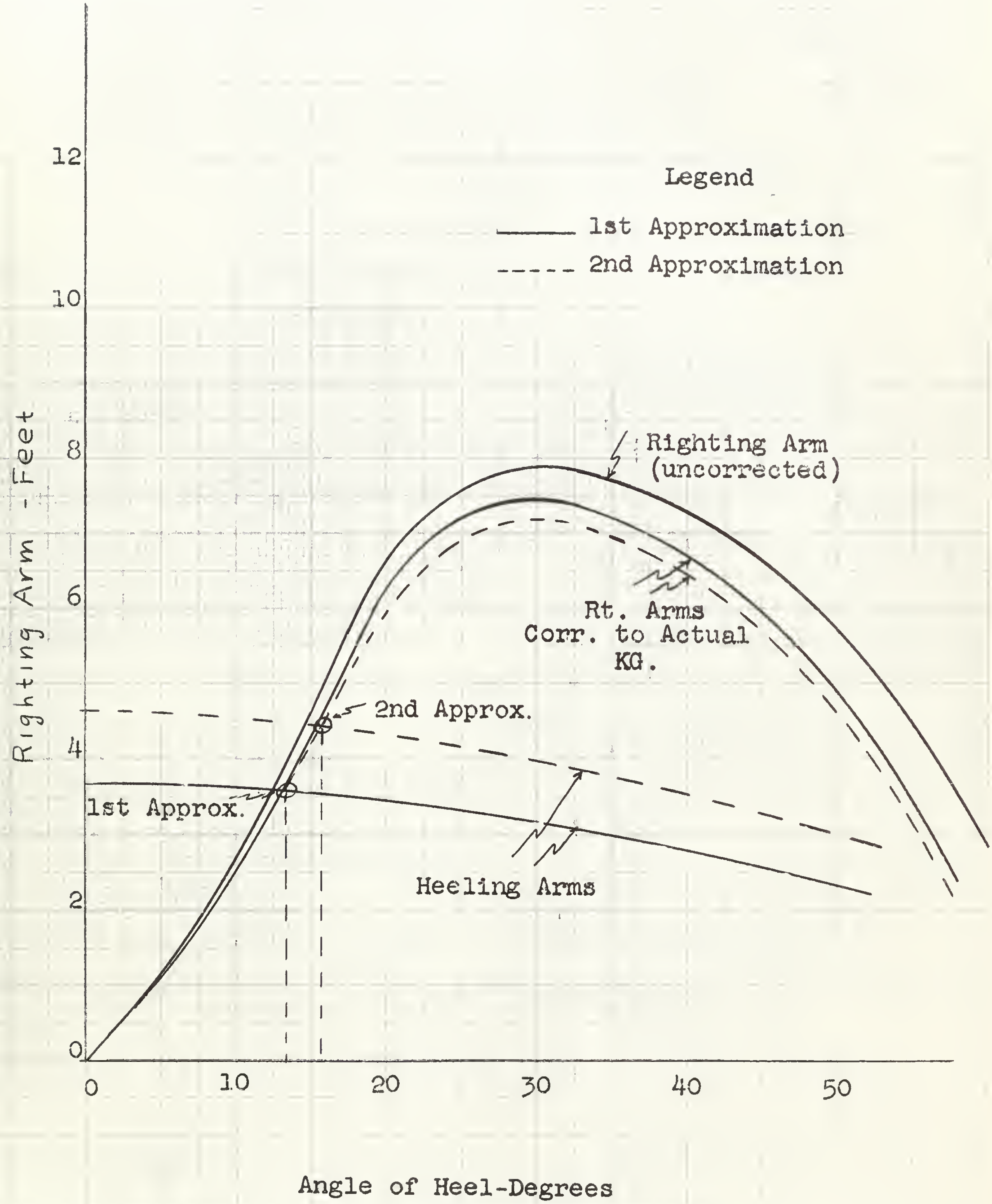


FIGURE XXXII

Curve of Righting Arm vs. Angle of Heel For Flooding Of Two Main Machinery Spaces

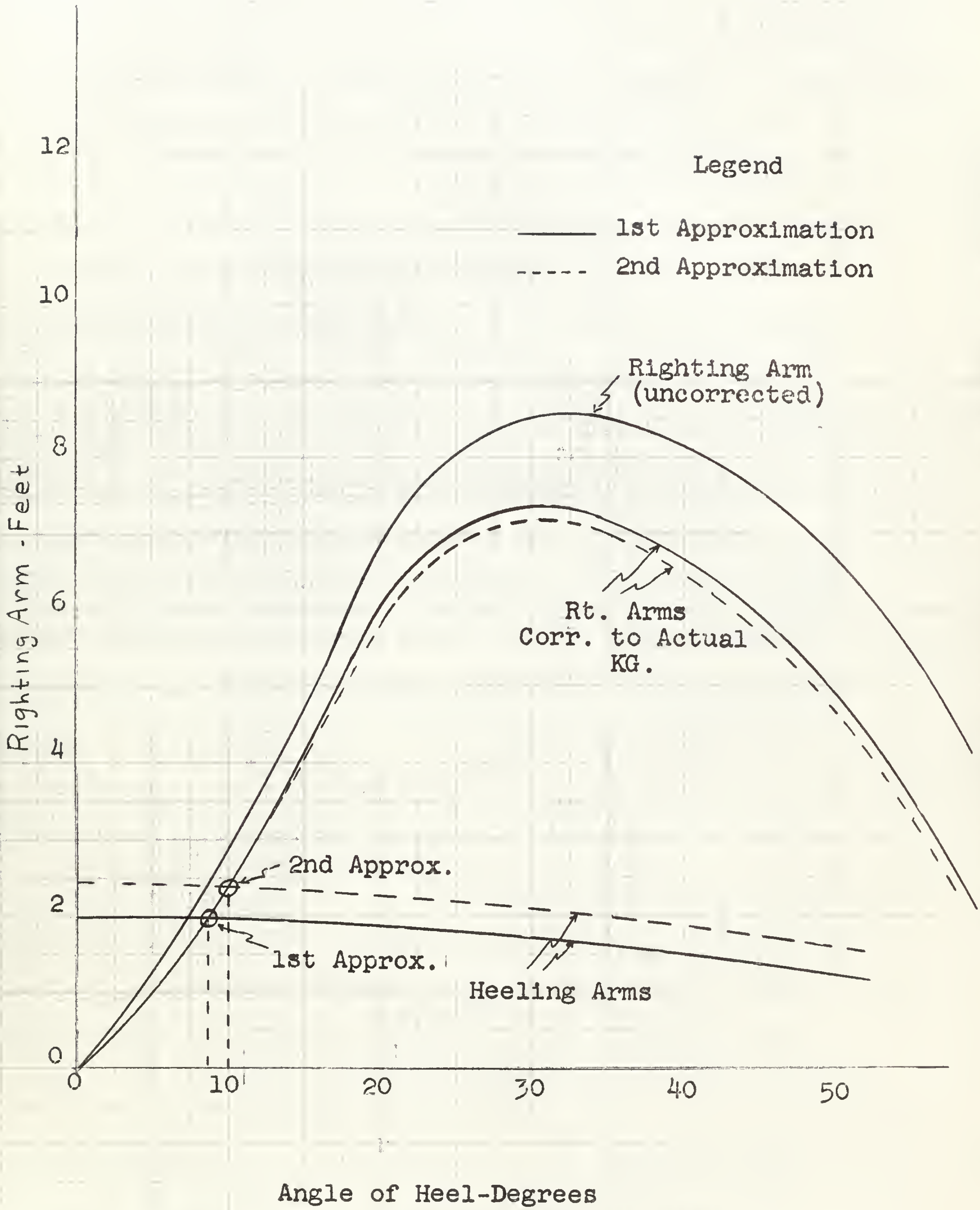


FIGURE XXXIII

Curve of Righting Arm vs. Angle Of Heel For Flooding Of Four Main Machinery Spaces

The calculated value of angle of list is slightly less than the value of the assumed list. However since they are almost equal, it can be assumed the ship will list to approximately 15.5° with four machinery spaces flooded and before counter-flooding is undertaken.

Preliminary calculations indicate that the 10° list resulting from the flooding of two machinery spaces could readily be removed by counter-flooding in the protective system on the opposite side of the ship. However for the 15.5° list resulting from flooding of four machinery spaces, counter-flooding would not be sufficient to remove the list entirely. In addition liquid would have to be removed on the damaged side of the ship from the liquid layers of the undamaged portion of the protective system.

APPENDIX F

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